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A METHOD FOR MULTI-CRITERIA DECISION MAKING UNDER UNCERTAINTY

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This work presents development of a method for multi-criteria decision making under uncertainty conditions based on single-attribute value functions and probabilistic distributions. The values of different criteria are modeled using normal distributions, i.e. the value of the i-th criteria of the j-th option is given by $x_i^j \sim \mathcal{N}(\mu_i^j, \sigma_i^j)$ distribution. The method is evaluated and the results are analyzed on a simple example.

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Introduction. Several evaluation methods have been proposed in literature to deal with multi-criteria decision making problems. Their general idea is the following. Suppose we have n criteria and m alternatives. For each criterion we are given a weight coefficient, k_i, which denotes the relative importance of the i-th criterion. For each of the alternatives, the decision makers determine values for all criteria. The i-th criterion for the j-th alternative is denoted by x_i^j. In Multi Attribute Value Theory (MAVT) we compute the values of single-attribute value functions, $u_i(x_i^j)$, where $u_i(x)$ is usually chosen to be a linear function depending on the minimum and maximum values of x [1]:

$$u_i(x_i^j) = \frac{x_i^j - x_i^{\min}}{x_i^{\max} - x_i^{\min}} \quad \text{or} \quad u_i(x_i^j) = \frac{x_i^{\max} - x_i^j}{x_i^{\max} - x_i^{\min}}. \tag{1}$$

At the last step of the method, the values $u(x^j) = \sum_{i=1}^{n} k_i u_i(x_i^j)$ are calculated for all x^j alternatives, which are used to solve the decision problem.

One issue of the described method is that it does not take into account the accuracy and correctness of the values given by decision makers. Sometimes the value estimations contain uncertainties and these uncertainties are different for different alternatives.

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Our Method. In order to measure the impact of the uncertainties on the final decisions, we suggest to replace the values of x_i^j with probabilistic distributions defined on the set of values for the given criterion. For simplicity, we suggest to model the values using normal distributions [2]. In particular, the value of the i-th criterion of the j-th alternative will be given by $x_i^j \sim \mathcal{N}(\mu_i^j, \sigma_i^j)$ distribution. Here μ_i^j correspond to the average, most probable value, while σ_i^j measures the uncertainty. Whenever the exact value of the criterion is known, we will set $\sigma_i^j = 0$.

The linear function $u_i(x)$, which is used to determine the single-attribute value function in MAVT method, will be applied to the distribution $\mathcal{N}(\mu_i^j, \sigma_i^j)$ instead of x_i^j scalar variable. This will ensure that the proportion of uncertainties will be conserved.

In particular:

when the goal is to maximize the i-th criterion, for $u_i(x)$ we have

$$u_i\left(\mathcal{N}\left(\mu_i^j, \sigma_i^j\right)\right) = \mathcal{N}\left(\frac{\mu_i^j - \mu_i^{\min}}{\mu_i^{\max} - \mu_i^{\min}}, \frac{\sigma_i^j}{\mu_i^{\max} - \mu_i^{\min}}\right), \tag{2}$$

when the goal is to minimize the i-th criterion, then

$$u_i\left(\mathcal{N}\left(\mu_i^j, \sigma_i^j\right)\right) = \mathcal{N}\left(\frac{\mu_i^{\max} - \mu_i^j}{\mu_i^{\max} - \mu_i^{\min}}, \frac{\sigma_i^j}{\mu_i^{\max} - \mu_i^{\min}}\right). \tag{3}$$

In these formulas $\mu_i^{\min} = \min_j \mu_i^j$ and $\mu_i^{\max} = \max_j \mu_i^j$. Note that the values of all single-attribute value functions are distributed normally, and the uncertainties of he values are proportional to the uncertainties given by decision makers.

We will apply the formula used in the last step of the MAVT method on the obtained normal distributions. The final value for the j-th alternative will be determined by the following formula:

$$u(x^j) = \sum_{i=1}^{n} k_i \mathcal{N}\left(\mu_i^j, \sigma_i^j\right). \tag{4}$$

Taking into account the following:

1. the values of single-attribute value functions are normal distributions;
2. the sum of normal distributions is also normal [3]:

$$\text{if } \; X \sim \mathcal{N}(\mu, \sigma),\, X' \sim \mathcal{N}(\mu', \sigma'), \; \text{ then } \; X + X' \sim \mathcal{N}(\mu + \mu', \sqrt{\sigma^2 + \sigma'^2}); \tag{5}$$

3. the product of a normal distribution and a scalar is also normal:

$$\text{if } \; X \sim \mathcal{N}(\mu, \sigma),\, k \in \mathbb{R}, \quad \text{then } \; kX \sim \mathcal{N}(k\mu, k\sigma), \tag{6}$$

we obtain that $u(x^j)$ is a normal distribution: $u(x^j) \sim \mathcal{N}(\mu^j, \sigma^j)$, where μ^j and σ^j depend on μ_i^j, σ_i^j and the form of u_i functions. Note that when $\sigma_i^j = 0$ for all values of i and j, then the described method is equivalent to MAVT method.

Let us derive the final form of the $u(x^j)$ distribution in a special case when we want to maximize the values of all criteria. Suppose we are given the numbers k_i, μ_i^j and σ_i^j, and all functions $u_i(x)$ are defined as in (2). Then, the formula (4) becomes:

$$u(x^j) = \sum_{i=1}^{n} k_i \mathcal{N}\left(\mu_i^j, \sigma_i^j\right) = \sum_{i=1}^{n} k_i \mathcal{N}\left(\frac{\mu_i^j - \mu_i^{\min}}{\mu_i^{\max} - \mu_i^{\min}}, \frac{\sigma_i^j}{\mu_i^{\max} - \mu_i^{\min}}\right) =$$
$$= \mathcal{N}\left(\sum_{i=1}^{n} k_i \frac{\mu_i^j - \mu_i^{\min}}{\mu_i^{\max} - \mu_i^{\min}}, \sqrt{\sum_{i=1}^{n} k_i \frac{\sigma_i^j}{\mu_i^{\max} - \mu_i^{\min}}}\right).$$

The Impact of Independence between Criteria. The formula (5) holds, if the random variables in the sum are independent. In reality, sometimes there can be some correlations between different criteria. In case the correlations are known, it is possible to extend our method in a way that the calculations will respect existing correlations. In particular, if the joint distribution of values of the i-th and i'-th criteria is normal and the correlation coefficient between the random variables $X \sim \mathcal{N}(\mu, \sigma)$ and $X' \sim \mathcal{N}(\mu', \sigma')$ is ρ, then the formula (5) will be modified as follows:

$$X + X' \sim \mathcal{N}(\mu + \mu', \sqrt{\sigma^2 + \sigma'^2 + 2\rho\sigma\sigma'}).$$

Application of the Proposed Method on a Simple Example. In [4], five scenarios of nuclear power unit installations are compared using MAVT method. The scenarios are WWER-1000, CANDU-6, small modular reactor having 360 *MW* power (SMR), ACP-600 and the non-nuclear option. The proposed method will be applied for five scenarios by taking into account economic criteria and the results will be analyzed. The economic criteria, taken into account in [4], are the following:

E.1 – Levelized Long-term average NPP production cost, LUEC;
E.2 – Power system Long-term average generation cost;
E.3 – New generation investment cost;
E.4 – Whole energy system cost.

Values for the economic criteria

Scenario	E.1, USD/$MW \cdot h$	E.2, USD/$MW \cdot h$	E.3, *mln* USD	E.4, *mln* USD
WWER-1000	91 (3)	75.9 (1)	8 566 (500)	44 555 (100)
CANDU-6	73 (3)	69.2 (1)	6 986 (500)	43 954 (100)
SMR (360 *MW*)	97 (3)	77.1 (1)	6 896 (1 000)	44 701 (200)
ACP-600	71 (3)	73.2 (1)	5 022 (1000)	44 347 (200)
No nuclear	103 (3)	78.3 (1)	2 431 (100)	44 868 (5)

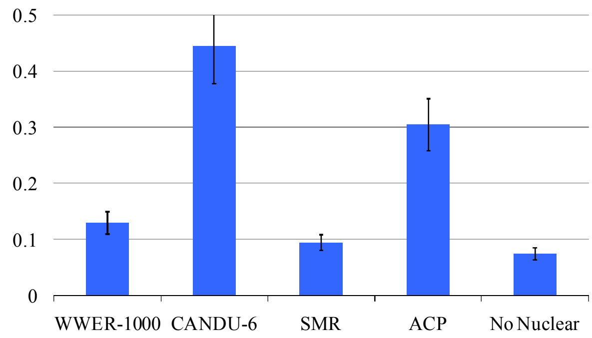
The values of the criteria considered in the table are taken from the "Long-term (up to 2036) development pathways of RA energy sector" strategy [5].

The values for the above-mentioned criteria are presented in Table. Taking into account the probability of changes of the values depending on different factors, we introduce uncertainty estimates σ_j^i, which are presented in Table inside parentheses.

The uncertainties presented in Table are based on the fact that American SMR and Chinese ACP-600 power units are still in design or testing phases. Therefore, the estimates of their costs are less precise than those of Canadian CANDU-6 and Russian WWER-1000, which are already in use. The costs for non-nuclear scenario are even more predictable.

Following [4], the weight coefficients for the criteria E.1 to E.4 are specified as follows: E.1 – 0.10; E.2 – 0.40; E.3 – 0.15; E.4 – 0.35.

Analysis of the Results. The results of applying our proposed method on this data are presented in Figure.



The final scores and uncertainties.

The obtained results demonstrate that Canadian CANDU-6 system significantly outperforms all rival technologies in terms of economic efficiency even by taking into account the uncertainties in data. The second most efficient system is the Chinese ACP-600, which, regardless the high uncertainty of its estimates, significantly outperforms WWER-1000 system, which comes at the third place. It is interesting to note the results for the American SMR system, which is the fourth in our list. The uncertainty in the data related to SMR technology is so high, that in case of the most positive developments it will become more preferable than WWER-1000 system, and in the case of the post negative developments it can be economically less efficient than the non-nuclear option. Also note, than there are scenarios when non-nuclear option can compete with WWER-1000 as well.

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