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*M e c h a n i c s*

# TRANSFER OF LOADS FROM A FINITE NUMBER OF ELASTIC OVERLAYS WITH FINITE LENGTHS TO AN ELASTIC STRIP THROUGH ADHESIVE SHEAR LAYERS

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This article deals with the problem of an elastic infinite strip, which is strengthened along its free boundary by a finite number of finite overlays with different elastic characteristics and small constant thicknesses. The interaction between the strip and the overlays is mediated by adhesive shear layers. The overlays are deformed under the action of horizontal forces. The problem of determination of unknown stresses acting between the strip and overlays are reduced to a system of Fredholm integral equations of the second kind for a finite number of unknown functions defined on different finite intervals. It is shown that in the certain domain of variation of the characteristic parameter of the problem this system of integral equations in Banach space may be solved by the method of successive approximations. Particular cases are discussed and the character and behaviour of unknown shear stresses are investigated.

**MSC2010:** 74M15.

***Keywords*:** contact, overlays (stringers), elastic infinite strip, adhesive shear layer, system of integral equations, operator equation.

**Introduction.** In [1,2] we considered the transfer of loads from a finite number of finite elastic overlays (stringers) to an elastic half-plane through adhesive layers. Still earlier, in [3,4] we considered a similar problem in the case of two finite elastic overlays for both the elastic half-plane and the infinite strip. The article [5] addresses the problem for an infinite plate (sheet) with two finite stringers, when only one of the stringers is connected through an adhesive layer. Also recall the papers [6–9], where different approaches were used to study similar problems for different elastic bodies, which are strengthened by a single finite stringer through an adhesive layer.

In this article we are going to consider the problem for an elastic infinite strip that is strengthened at finite intervals of its free boundary by an arbitrary finite number of finite overlays with different elastic moduli and small constant thicknesses.

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**Statement of the Problem and Obtaining the System of Integral Equations.** Let an elastic infinite strip (plane deformation, with Young's modulus $E$, Poisson's ratio $\nu$ and thickness $H$) be rigidly attached to non-deformable foundation at its boundary $y=-H$ and strengthened along its free boundary $y=0$ at the finite $[a_j,b_j]$ ($b_j>a_j$, $b_j<a_{j+1}$) intervals by a finite number of finite overlays (stringers) of small constant thicknesses $h_j$ ($h_j \ll b_j-a_j$) and with elastic moduli $E_j$ (*hereinafter, the index j takes values in the range 1 to n, unless otherwise stated*). The interaction between the strip and the overlays is mediated by thin uniform elastic adhesive layers with Young's modulus $E_k$, Poisson's ratio $\nu_k$ and small constant thickness $h_k$. The problem is to determine the distribution law of unknown stresses acting between the strip and the overlays, when concentrated forces $P_j$, are applied at one end points of overlays $x=b_j$, and directed along the $Ox$ axis (see Figure).

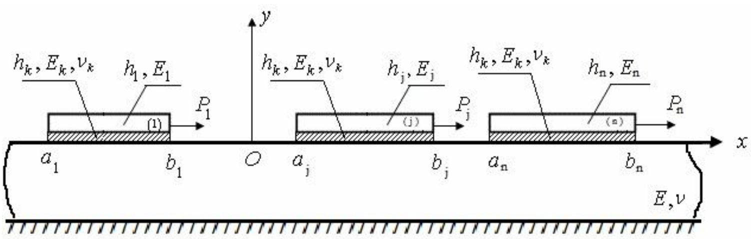


Figure.

The model of uniaxial strain state and the conditions of pure shear are assumed for the overlays (stringers) [10] and the adhesive layers [6], respectively. It is also assumed that only shear (tangential) stresses act between the strip and the overlays [1–9].

With such assumptions, let write the horizontal displacements $u(x,0)$ of the boundary points of the elastic strip in the form [3]:

$$u(x,0)=\sum_{i=1}^{n}\int_{a_i}^{b_i} K(|x-s|)\,\tau_i(s)\,ds, \tag{1}$$

when on the $[a_j,b_j]$ segments of its free $y=0$ boundary there act shear (tangential) forces with intensities $\tau_j(x)$, respectively. In Eq. (1)

$$K(|x|)=\frac{1}{2\pi}\int_{-\infty}^{\infty} K_n(\sigma)e^{-i\sigma x}\,d\sigma,$$

$$K_n(\sigma)=\frac{(2\chi+1)[(\chi+1)\sinh 2H|\sigma|+2\chi H|\sigma|]}{2\mu|\sigma|[2\chi(\chi+1)\cosh 2H\sigma+\chi^2(4H^2\sigma^2+1)+(\chi+1)^2]}, \tag{2}$$

$$\chi=\frac{\lambda+\mu}{2\mu},$$

$\lambda$, $\mu$ are Lame's constants of elasticity of the material of the strip.

Now, assuming that each differential element of the adhesive layers is in a condition of pure shear [1–9], the following conditions are obtained:

$$u^{(j)}(x)-u(x,0)=k\tau_j(x), \quad a_j\leqslant x\leqslant b_j, \quad j=\overline{1,n}, \tag{3}$$

where $k = h_k/G_k$, $G_k = E_k/2(1+\nu_k)$, $\tau_j(x) = G_k\gamma_k^{(j)}(x)$. $G_k$ is the shear modulus of the adhesive layers, $u^{(j)}(x)$ are the horizontal displacements of the points of the elastic overlays on the $[a_j,b_j]$ intervals, $\tau_j(x)$ and $\gamma_k^{(j)}(x)$ are the shear stresses and the shear strains in the adhesive layers on the $[a_j,b_j]$ intervals, respectively.

Further, taking into account the above assumptions [1–10], the equilibrium differential equations for overlays on the $[a_j,b_j]$ finite intervals will be written in the following form:

$$\frac{d^2u^{(j)}}{dx^2} = \frac{\tau_j(x)}{E_jh_j}, \quad a_j \leqslant x \leqslant b_j, \quad j = \overline{1,n}, \tag{4}$$

which, by virtue of (3), can be written in the form:

$$\frac{d^2u^{(j)}}{dx^2} - \gamma_j^2 u^{(j)}(x) = -\gamma_j^2 u(x,0), \quad a_j \leqslant x \leqslant b_j, \tag{5}$$

where we have also the following boundary conditions:

$$\left.\frac{du^{(j)}}{dx}\right|_{x=a_j} = 0, \quad \left.\frac{du^{(j)}}{dx}\right|_{x=b_j} = \frac{P_j}{E_jh_j}. \tag{6}$$

Here $\gamma_j^2 = 1/kE_jh_j$, $\quad j = \overline{1,n}$.

The solutions of the boundary value problems (5) and (6) we obtaim in the form:

$$u^{(j)}(x) = u_0^{(j)}(x) + \gamma_j^2 \int_{a_j}^{b_j} G_j(x,s)u(s,0)\,ds, \quad a_j \leqslant x \leqslant b_j, \tag{7}$$

where $u_0^{(j)}(x)$ are the general solutions of the homogenous equations corresponding to Eq. (5) with the boundary conditions (6) and have the form

$$u_0^{(j)}(x) = \frac{P_j \cosh[\gamma_j(x-a_j)]}{\gamma_jE_jh_j \sinh[\gamma_j(b_j-a_j)]},$$

and $G_j(x,s)$ are Green's functions [11]:

$$G_j(x,s) = \frac{1}{\gamma_j \sinh[\gamma_j(b_j-a_j)]} \begin{cases} \cosh\gamma_j(x-b_j)\cosh\gamma_j(s-a_j), & x > s, \\ \cosh\gamma_j(x-a_j)\cosh\gamma_j(s-b_j), & x < s. \end{cases}$$

It is obvious, that the functions $G_j(x,s)$ are continuous in both variables and are symmetric with respect to permutation of variables $G_j(x,s) = G_j(s,x)$.

Further, by virtue of (7), from (3) we obtain the following equations

$$k\tau_j(x) + u(x,0) = \gamma_j^2 \int_{a_j}^{b_j} G_j(x,s)u(s,0)\,ds + u_0^{(j)}(x), \quad a_j \leqslant x \leqslant b_j, \quad j = \overline{1,n}. \tag{8}$$

Now, by matching (1) and (8), we get a system of integral equations with respect to unknown shear stresses $\tau_j(x)$, which are specified in different finite $[a_j,b_j]$

intervals, in the form:

$$\tau_j(x)+\frac{1}{k}\sum_{i=1}^{n}\int_{a_i}^{b_i}K(|x-s|)\tau_i(s)\,ds-$$
$$-\frac{\gamma_j^2}{k}\sum_{i=1}^{n}\int_{a_j}^{b_j}G_j(x,s)\left[\int_{a_i}^{b_i}K(|s-t|)\tau_i(t)dt\right]ds=\frac{u_0^{(j)}(x)}{k},\quad j=\overline{1,n},\quad (9)$$

or, changing the order of integration in double integrals, in the form:

$$\tau_j(x)+\frac{1}{k}\sum_{i=1}^{n}\int_{a_i}^{b_i}K(|x-t|)\tau_i(t)\,dt-$$
$$-\frac{\gamma_j^2}{k}\sum_{i=1}^{n}\int_{a_i}^{b_i}\left[\int_{a_j}^{b_j}G_j(x,s)K(|s-t|)ds\right]\tau_i(t)dt=\frac{u_0^{(j)}(x)}{k}.\quad (10)$$

Further, note that according to (2) the kernel $K(|x|)$ can be represented in the following form:

$$K(|x|)=\frac{A_1}{\pi}\left(\ln\frac{1}{|A_1x|}-C\right)+\frac{A_1}{\pi}R(x),\quad (11)$$

where $A_1=\dfrac{2\chi+1}{4\chi\mu}=\dfrac{2(1-\nu^2)}{E}$ and $C$ is Euler's constant. In (11) $R(x)$ is the regular part of the kernel $K(|x|)$ and has the form

$$R(x)=\pi\sum_{n=1}^{\infty}(-1)^n\left[\frac{(A_1x)^{2n}}{\pi(2n)!}\left(\ln\frac{1}{|A_1x|}+1+\frac{1}{2}+\cdots+\frac{1}{2n}-C\right)-\frac{|A_1x|^{2n-1}}{2(2n-1)!}\right]+$$
$$+\frac{1}{2A_1}\int_{-\infty}^{\infty}\frac{[(A_1+|\sigma|)K_n(\sigma)-A_1]e^{-i\sigma x}\,d\sigma}{A_1+|\sigma|},$$

since

$$K_n(\sigma)\sim A_1|\sigma|^{-1},\quad \text{as}\quad |\sigma|\to\infty.\quad (12)$$

Now, by virtue of (11), we can bring the system (10) to the form:

$$\tau_j(x)+\frac{A_1}{\pi k}\sum_{i=1}^{n}\int_{a_i}^{b_i}\left(\ln\frac{1}{A_1|x-t|}-C\right)\tau_i(t)\,dt+\frac{A_1}{\pi k}\sum_{i=1}^{n}\int_{a_i}^{b_i}R(x-t)\tau_i(t)\,dt-$$
$$-\frac{\gamma_j^2A_1}{\pi k}\sum_{i=1}^{n}\int_{a_i}^{b_i}\left[\int_{a_j}^{b_j}G_j(x,s)\left(\ln\frac{1}{A_1|s-t|}-C\right)ds\right]\tau_i(t)\,dt-\quad (13)$$
$$-\frac{\gamma_j^2A_1}{\pi k}\sum_{i-1}^{n}\int_{a_i}^{b_i}\left[\int_{a_j}^{b_j}G_j(x,s)R(s-t)\,ds\right]\tau_i(t)\,dt=\frac{u_0^{(j)}(x)}{k}.$$

Next, replacing the variables $x$, $s$ and $t$ by $ax$, $as$ and $at$ respectively, where $a > 0$ is the coordinate of one of the endpoints of a certain overlay, we obtain the system (13) in the following form:

$$\psi_j(x) + \delta^2 \sum_{i=1}^{n} \int_{\alpha_i}^{\beta_i} \ln \frac{1}{|x-t|} \psi_i(t)\,dt + \delta^2 \sum_{i=1}^{n} \int_{\alpha_i}^{\beta_i} R_*(x-t)\,\psi_i(t)\,dt -$$

$$- a\gamma_j^2 \delta^2 \sum_{i=1}^{n} \int_{\alpha_i}^{\beta_i} \left[ \int_{\alpha_j}^{\beta_j} G_j(ax,as) \ln \frac{1}{|s-t|} ds \right] \psi_i(t)\,dt -$$

$$- a\gamma_j^2 \delta^2 \sum_{i=1}^{n} \int_{\alpha_i}^{\beta_i} \left[ \int_{\alpha_j}^{\beta_j} G_j(ax,as) R_*(s-t)\,ds \right] \psi_i(t)\,dt = \frac{a u_0^{(j)}(ax)}{k}. \quad (14)$$

One can represent the system of integral equations (14) in the following form:

$$\psi_j(x) + \delta^2 \sum_{i=1}^{n} \int_{\alpha_i}^{\beta_i} L_j(x,t)\psi_i(t)\,dt = f_0^{(j)}(x), \quad \alpha_j \leqslant x \leqslant \beta_j, \quad j = \overline{1,n}, \quad (15)$$

where

$$L_j(x,t) = \ln \frac{1}{|x-t|} + R_*(x-t) -$$

$$- a\gamma_j^2 \int_{\alpha_j}^{\beta_j} G_j(ax,as) \ln \frac{1}{|s-t|} ds - a\gamma_j^2 \int_{\alpha_j}^{\beta_j} G_j(ax,as) R_*(s-t)\,ds, \quad (16)$$

$$f_0^{(j)}(x) = \frac{a u_0^{(j)}(ax)}{k} = \frac{P_j a\gamma_j \cosh[a\gamma_j(x-\alpha_j)]}{\sinh[a\gamma_j(\beta_j-\alpha_j)]},$$

$$\delta^2 = aA_1/\pi k, \ \alpha_j = a_j/a, \ \beta_j = b_j/a, \ \psi_j(x) = a\tau_j(ax)\,, R_*(z) = R(az),$$

$R_*(z)$ is a square integrable function.

Note that the system (10) or (15) has been obtained by the changing the order of integration, the validity of which follows from the Fubini's theorem [11], as well as from the equalities

$$\int_{\alpha_j}^{\beta_j} G_j(ax,as)\,ds = \frac{1}{a\gamma_j^2}, \quad (17)$$

which, in turn, follow from the equalities [1,2]:

$$\int_{a_j}^{b_j} G_j(x,s) \cos\left[\frac{m\pi(s-a_j)}{b_j-a_j}\right] ds =$$

$$= \frac{(b_j-a_j)^2}{(b_j-a_j)^2\gamma_j^2 + m^2\pi^2} \cos\left[\frac{m\pi(x-a_j)}{b_j-a_j}\right], \quad m = 0,1,2,\ldots, \quad (18)$$

where the functions $\cos\left[\dfrac{m\pi(x-a_j)}{b_j-a_j}\right]$, $m=0,1,2,\ldots$, form full orthogonal systems in the spaces $L_2(a_j,b_j)$. Note [11] that the symmetric completely continuous integral operator $B$:

$$B\varphi=\int\limits_a^b G(x,s)\varphi(s)\,ds,$$

which acts in the space $L_2(a,b)$, is an inverse of the operator $D=-d^2/dx^2+\gamma^2 I$.

Below the Fubini's theorem will be used without special mention.

Also note that the system (15) has been obtained without using the equilibrium conditions imposed on the overlays

$$\int\limits_{\alpha_j}^{\beta_j}\psi_j(ax)\,dx=P_j. \tag{19}$$

In the system (15), the conditions (19) are satisfied automatically due to the following relations

$$\int\limits_{\alpha_j}^{\beta_j} f_0^{(j)}(x)\,dx=P_j/a.$$

These can be easily verified by integrating the equations (15) from $\alpha_j$ to $\beta_j$, then changing the order of integration in resulting double integrals and taking into account the equalities

$$\int\limits_{\alpha_j}^{\beta_j} L_j(x,t)\,dx=0,$$

which follow from (18).

Thus, the solution of the problem is reduced to solving the system (15) of Fredholm integral equations of the second kind with kernels squarely integrable in two variables and with right-hand sides which are the solutions of the problem in the case of rigid base.

From the system (15), it is easy to see that at the end points of overlays $x=\alpha_j$ and $x=\beta_j$ the values of unknown shear stresses $\psi_j(x)$ are finite.

**Investigation of Solvability of the system of Integral Equations** (15)**.** Further, write system (15) in the following form:

$$\varphi+T\,\varphi=g_0, \tag{20}$$

where

$$\varphi=\begin{pmatrix}\psi_1\\ \psi_2\\ \vdots\\ \psi_n\end{pmatrix},\quad g_0=\begin{pmatrix}f_0^{(1)}\\ f_0^{(2)}\\ \vdots\\ f_0^{(n)}\end{pmatrix},\quad T=\begin{pmatrix}\delta^2k_{11} & \delta^2k_{12} & \ldots & \delta^2k_{1n}\\ \delta^2k_{21} & \delta^2k_{22} & \ldots & \delta^2k_{2n}\\ \vdots & & & \\ \delta^2k_{n1} & \delta^2k_{n2} & \ldots & \delta^2k_{nn}\end{pmatrix},$$

$$k_{ji}\psi_i = \int\limits_{\alpha_i}^{\beta_i} L_j(x,t)\psi_i(t)\,dt, \quad j,i = \overline{1,n}. \tag{21}$$

Now consider Eq. (20) in Banach space by dint of vector-functions $X = \begin{pmatrix} X_1 \\ X_2 \\ \vdots \\ X_n \end{pmatrix}$, where $X_j \in L_2(\alpha_j,\beta_j)$, and with the norm:

$$\|X\| = \max\left\{\|X_1\|_{L_2(\alpha_1,\beta_1)}, \|X_2\|_{L_2(\alpha_2,\beta_2)}, \ldots, \|X_n\|_{L_2(\alpha_n,\beta_n)}\right\}.$$

$L_2(\alpha_j,\beta_j)$ are spaces of square integrable functions, specified on the intervals $(\alpha_j,\beta_j)$.

Operators $k_{ji},\ j,i=\overline{1,n}$, act as follows: $k_{ji}: L_2(\alpha_i,\beta_i) \to L_2(\alpha_j,\beta_j)$.

Obviously, the operator $T$ acts in the $B$ space and is Fredholm's operator. The operational Eq. (20) in the $B$ space can be solved by the method of successive approximations, if $\|T\| < 1$, where

$$\|T\| = \max\left\{\delta^2\left(\|k_{11}\| + \cdots + \|k_{1n}\|\right), \ldots, \delta^2\left(\|k_{n1}\| + \cdots + \|k_{nn}\|\right)\right\}.$$

That is, the condition $\|T\| < 1$ will be satisfied, if

$$\delta^2\left(\|k_{11}\| + \cdots + \|k_{1n}\|\right) < 1,\ \delta^2\left(\|k_{21}\| + \cdots + \|k_{2n}\|\right) < 1, \ldots, \\ \delta^2\left(\|k_{n1}\| + \cdots + \|k_{nn}\|\right) < 1. \tag{22}$$

Then the solution of Eq. (20) will be written in the form:

$$\varphi = (I+T)^{-1}g_0 = \sum_{m=0}^{\infty}(-1)^m T^m g_0.$$

Now determine the values of $\delta^2$ parameter, for which the conditions (22) will be satisfied. From (21), by virtue of Cauchy–Bunyakovski inequality, we get:

$$\left\|k_{ji}\right\| \leqslant e_{ji}, \quad e_{ji} = \left(\int\limits_{\alpha_i}^{\beta_i}\int\limits_{\alpha_j}^{\beta_j} L_j^2(x,t)\,dx\,dt\right)^{\frac{1}{2}}, \quad j,i = \overline{1,n}. \tag{23}$$

The expressions for $e_{ji}$, are difficult to calculate, so, we will estimate them. As will be justified below, the following estimates take place:

$$e_{ji} < \frac{l_j}{2}\left(\int\limits_{\alpha_i}^{\beta_i}\int\limits_{\alpha_j}^{\beta_j} \ln^2|x-t|\,dx\,dt\right)^{\frac{1}{2}} + \frac{l_j}{2}\left(\int\limits_{\alpha_i}^{\beta_i}\int\limits_{\alpha_j}^{\beta_j} R_*^2(x-t)\,dxdt\right)^{\frac{1}{2}}, \ l_j = \beta_j - \alpha_j. \tag{24}$$

First, let us get the estimate (24) in the case of $i = j$. To this end, recall that in the Eq. (18), as mentioned above, the functions $\cos[m\pi(x-\alpha_j)/l_j],\ m = 0,1,2,\ldots$, constitute full orthogonal systems in the spaces $L_2(\alpha_j,\beta_j)$. Then, according to Parseval's equality, we have (since $L_{j0} = 0$)

$$\frac{2}{l_j}\int\limits_{\alpha_j}^{\beta_j} L_j^2(x,t)\,dx = \sum_{m=1}^{\infty} L_{jm}^2(t), \quad \alpha_j < t < \beta_j, \tag{25}$$

where

$$L_{jm}(t) = \int\limits_{\alpha_j}^{\beta_j} L_j(x,t)\cos\left[\frac{m\pi(x-\alpha_j)}{l_j}\right]dx, \quad m = 1,2,\ldots$$

Further, according to (18), we have

$$L_{jm}(t) = \left(1 - \frac{\gamma_1^2 l_j^2}{\gamma_1^2 l_j^2 + m^2\pi^2}\right)\eta_{jm}(t), \quad m = 1,2,\ldots,$$

where

$$\eta_{jm}(t) = \int\limits_{\alpha_j}^{\beta_j}\left[\ln\frac{1}{|x-t|} + R_*(x-t)\right]\cos\left[\frac{m\pi(x-\alpha_j)}{l_j}\right]dx, \; m = 1,2,\ldots$$

Therefore,

$$\frac{2}{l_j}\int\limits_{\alpha_j}^{\beta_j} L_j^2(x,t)\,dx = \sum_{m=1}^{\infty}\left(1 - \frac{\gamma_j^2 l_j^2}{\gamma_j^2 l_j^2 + m^2\pi^2}\right)^2 \eta_{jm}^2(t) < \sum_{m=1}^{\infty}\eta_{jm}^2(t),$$

$$\alpha_j < t < \beta_j.$$

On the other hand, by virtue Cauchy–Bunyakovski inequality, we obtain

$$\sum_{m=1}^{\infty}\eta_{jm}^2(t) \le \frac{l_j}{2}\int\limits_{\alpha_j}^{\beta_j}\left[\ln\frac{1}{|x-t|} + R_*(x-t)\right]^2 dx.$$

Finally, applying the Cauchy inequality, we get:

$$e_{jj} < \frac{l_j}{2}\left(\int\limits_{\alpha_j}^{\beta_j}\int\limits_{\alpha_j}^{\beta_j}\ln^2|x-t|\,dx\,dt\right)^{\frac{1}{2}} + \frac{l_j}{2}\left(\int\limits_{\alpha_j}^{\beta_j}\int\limits_{\alpha_j}^{\beta_j} R_*^2(x-t)\,dx\,dt\right)^{\frac{1}{2}}.$$

The estimates (24) for $e_{ji}$, $j \ne i$, one can obtain in a similar way.

Then the conditions (22) will be satisfied if

$$\delta^2 < \left(\sum_{i=1}^{n} e_{j_i}\right)^{-1} = e_j. \tag{26}$$

Therefore, we have obtained the condition of realization of (22) in the form $\delta^2 < \min(e_1, e_2, \ldots, e_n)$, where $e_j$ are positive numbers, less than unity.

The values of unknown shear stresses $\psi_j(x)$ at the end points $x = \alpha_j$ and $x = \beta_j$ of overlays one can obtain from system (15).

It may be appropriate to note that comprehensive and deep numerical analysis for the elastic half-plane in cases of $n = 1,2$ are presented in [1]. Without going into details, note that some still unpublished numerical results of analyzing the system of integral equations (15) with $n = 1,2$ in some particular cases with the matching values of the characteristic parameters correspond to the calculated results presented in [1] (see Figs. 4, 5 and Fig. 7 in [1]).

**Conclusion.** To study the changes in the law of distribution of unknown shear stresses depend on the characteristic parameters of the problem, this article presents an effective solution to the problem in question. The problem is reduced to solving a finite system of Fredholm integral equations of the second kind, which can be solved by the method of successive approximations. The solution of the problem in question can also be reduced to solving a finite system of singular integro-differential equations of the second kind with a Cauchy kernel and with certain boundary conditions. Its solutions can be constructed using Chebishev's orthogonal polynomials of second kind using the method given in [3].

*A brief account on the presented work has been reported at the IX International Conference on the problem of dynamics of interaction of deformable media [12].*

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Ա. Վ. ՔԵՐՈԲՅԱՆ

## ՎԵՐՋԱՎՈՐ ԹՎՈՎ ՎԵՐՋԱՎՈՐ ՎԵՐԱԴԻՐՆԵՐԻՑ ԿՊՉՈՒՆ ՍԱՀՔԻ ՇԵՐՏԵՐԻ ՄԻՋՈՑՈՎ ԲԵՌՆԱՎՈՐՈՒՄՆԵՐԻ ՓՈԽԱՆՑՈՒՄՆ ԱՌԱՁԳԱԿԱՆ ՇԵՐՏԻՆ

Դիտարկված է առաձգական անվերջ շերտի վերաբերյալ խնդիր, որն իր ազատ եզրի երկայնությամբ ուժեղացված է տարբեր առաձգական հատկություն-ներով և փոքր հաստատուն հաստություններով, վերջավոր թվով վերջավոր երկարությամբ վերադիրներով: Առաձգական շերտի և վերադիրների միջև փոխազդեցությունն իրագործվում է միատեսակ կպչուն շերտերի միջոցով: Վերադիրները դեֆորմացիայի են ենթարկվում նրանց ծայրերում կիրառված հորիզոնական ուժերի ազդեցության տակ: Շերտի և վերադիրների միջև գործող անհայտ շոշափող լարումների որոշման խնդիրը հանգեցված է տարբեր միջա-կայքերում որոշված վերջավոր թվով անհայտ ֆունկցիաների նկատմամբ Ֆրեդհոլմի երկրորդ սեռի ինտեգրալ հավասարումների համակարգի լուծմանը: Ցույց է տրված, որ խնդրի բնութագրիչ պարամետրերի փոփոխության որոշակի տիրույթում, ինտեգրալային հավասարումների այս համակարգը Բանախի տարածության մեջ կարող է լուծվել հաջորդական մոտավորությունների եղանակով: Դիտարված են մասնավոր դեպքեր և պարզաբանված է շոշափող լարումների բաշխման բնույթը և վարքը: