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OPTIMAL CONTROL OF DYNAMIC SEARCH OF A MOVING OBJECT IN A RECTANGULAR DOMAIN

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The problem of optimal control of the spatial motion of a dynamic object in order to find a moving object performing a simple motion in a rectangular domain on the plane is considered. A method for controlling the motion of the searching object, as well as the corresponding law of variation of the electric current in the light source circuit, ensuring the detection of the sought object with minimal light energy consumption for a guaranteed search time, are proposed.

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Introduction. In many problems of search for target object, detection is performed using the information area of sensitivity [1]. As such, it is possible to consider an area illuminated by a light source that can be moved in space in order to detect the desired object when it enters this area [2]. In the case of a moving object in a limited area, an approach is used to solve the search problem [3], which consists in constructing controls in such a way that moving along the corresponding trajectories, searching object performs a scan by sweeping the bands covering the entire search area. Under certain conditions on the search engine parameters, this approach determines the set of controls that guarantee the successful completion of the search for a target object, both mobile [4, 5] and immobile [6, 7]. In this regard, it is advisable to consider the problem of the optimal choice of guaranteeing control. As an optimality criterion, a functional is considered that takes into account power inputs of the light source located on the searching object. Unlike [1–7], in this paper searching object is controlled by acceleration, and the lighting area is square. A method for controlling the motion of the searching object, as well as the corresponding law of variation of the electric current in the light source circuit, ensuring the detection of the sought object with minimal light power input for a guaranteed search time, are proposed.

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Statement of the Problem. Consider a system of two controlled point objects X (searching) and Y (sought), whose motion is described by the following equations, initial conditions and constraints:

$$\begin{gathered} X : \ddot{x}_1 = w_1, \quad \ddot{x}_2 = w_2, \quad \ddot{x}_3 = w_3 - g, \\ x_i(0) = x_i^0, \quad \dot{x}_i(0) = 0, \quad i = 1,2,3, \\ |w_1(t)| \leq W, \quad |w_2(t)| \leq W, \quad |w_3(t)| \leq W_3, \quad W_3 > g, \\ x(t) \in D^{(3)} = \{(x_1,x_2,x_3) : \ 0 \leq x_i \leq a_i, \ i = 1,2,3\}, \end{gathered} \tag{1}$$

$$\begin{gathered} Y : \dot{y}_i = v_i, \quad y_i(0) = y_i^0, \quad i = 1,2, \quad \sqrt{v_1^2 + v_2^2} \leq V, \\ y(t) \in D^(2) = \{x_1, x_2\,() : 0 \leq x_i \leq a_i, \ i = 1,2\}, \quad t \geq 0. \end{gathered} \tag{2}$$

In (1), (2) x_i, y_i are geometrical coordinates of the objects X,Y; w_i, v_i are components of controlled acceleration w and controlled speed v of objects X,Y, which are piecewise continuous functions of time; W, W_3, V, a_i are given constants, g is gravitational acceleration.

Let us assume that all information on the parameters and relations (1), (2), except for the initial state $y(0) = \left(y_1^0, y_2^0\right)$ and the current velocity $v(t)$ of the object Y, is available to the controlled object X. Suppose that to determine the exact coordinates of Y, the object X has a special device in the form of a regular quadrangular pyramid, at the apex of which an isotropic point light source is located. The light rays emitted from the source are limited inside the pyramid, as a result of which, on the horizontal search plane, a moving and varying in size area of illumination of the following type is formed:

$$\begin{gathered} K(x(t)) = \left\{(\zeta_1, \zeta_2) \in D^{(2)} : |\zeta_{1,2} - x_{1,2}(t)| \leq l = Cx_3(t), \quad C = \tan\gamma/\sqrt{2}\right\}, \\ x(t) \in D^{(3)}. \end{gathered} \tag{3}$$

Domain (3) is a square with the center at the point $O(x_1(t), x_2(t)) \in D^{(2)}$ and side length $2l$; γ, $0 < \gamma < \pi/2$ is half the opening angle of light rays emanating from a point source and forming opposite edges of the pyramid; x_3 is the distance from the top of the pyramid to the center of the square.

Object X detects object Y at some instant of time $t > 0$, if the following search condition is met:

$$y(t) \in K(x(t)). \tag{4}$$

The sought object Y, if it falls into the lighted region (3), can be detected or recognized only in the case of sufficient constant illumination E characterizing the threshold value of the visibility of the sought object.

According to [8], in the case of (3) the minimum sufficient illumination at a certain point on the plane is calculated by the

$$E_P = I\cos\gamma/(SP)^2, \tag{5}$$

where I is the light intensity of the source S in the direction of the measuring point P on the plane; SP is the distance between the light source and this point; γ is the angle between the direction of light incidence and the perpendicular to this plane.

From (5) it follows that for a square region (3) for given x_3 and γ the illumination is maximal at the point closest to the source at the center of the square:

$E_O = E_{\max} = I/x_3^2$ and minimal at the most distant point at the corner points of the square:

$$E = E_{\min} = \xi I/x_3^2, \quad \xi = \cos\gamma/(1+\tan^2\gamma). \tag{6}$$

The value $E = E_{\min}$ (6) will be considered constant and given.

We use the relation $Q = \eta I$ [8]. Here Q is the power of light energy, which can be considered equal to the electrical power consumed by the light source; I is the luminous intensity; η is the coefficient of proportionality (power density factor). Then the value of the minimum illumination E (6) can be calculated as the power of the energy of the light radiation incident on the plane:

$$E = \xi I/x_3^2 = \xi Q/\eta x_3^2. \tag{7}$$

The integral of function Q from (7) with $E,\gamma = \text{const},\ 0 < \gamma < \pi/2$,

$$J = \int_0^T Q dt = E\eta\xi^{-1}\int_0^T x_3^2 dt \tag{8}$$

gives the energy consumed by the light source during $[0,T]$.

Functional (8) characterizes power input in the course of search by the light device and, according to (1), it is a function of control w_3. The electric energy consumed by the light source during the illumination time interval $[0,T]$ can be expressed as

$$J = \int_0^T Q dt = \int_0^T j^2 R dt,\ \ Q = j^2 R,\ \ 0 \le j(t) \le j_0,\ \ t \in [0,T], \tag{9}$$

where j is the actual value of the current through the light source; j_0 is the maximal admissible value of actual current and R is the active resistance in the light source circuit.

From (7), (9) we obtain the dependence of the electric current j on the distance x_3 from the point-like light source to the center of the light domain:

$$\begin{gathered} j(t) = \sqrt{E\eta\xi^{-1}R^{-1}}x_3(t),\ \ x_3(t) > 0, \\ 0 \le j(t) \le \min\left(j_0\sqrt{E\eta\xi^{-1}R^{-1}a_3}\right),\ \ t \in [0,T]. \end{gathered} \tag{10}$$

Relation (10), taking into account third equation of (1), determines the relationship between the functions $w_3 = w_3(t)$ and $j = j(t)$.

P r o b l e m . *Find an initial position $x^0 = \left(x_1^0, x_2^0, x_3^0,\right) \in D^{(3)}$, number $T > 0$, an admissible control $w(t)$ of object X on interval $[0,T]$ and corresponding law of electric current variation in the light source circuit $j = j(t)$, $t \in [0,T]$ for which, at any initial position $y^0 = \left(y_1^0, y_2^0\right) \in D^{(3)}$ and any admissible control $v(t)$ of object Y on $[0,T]$, it is guaranteed that condition (4) is satisfies at some instant in $[0,T]$ with minimal light energy consumption (8).*

Description of the Search Method. At first we describe the control method being proposed, and then indicate the conditions on the parameters entering into it, under which the Problem can be solved. Let at the initial instant $t = 0$ object X is at the point $x^0 = \left(x_1^0, x_2^0, x_3^0,\right)$, $x_1^0 = x_2^0 = l_0,\ x_3^0 = C^{-1}l_0,\ 0 < x_3^0 \le a_3,$ where $l_0 \le Ca_3 < a_2/2,\ a_2 = \min(a_1, a_2)$. Consider the spatial broken line outgoing this point, whose projection onto the rectangular base $D^{(2)}$ is $L_{0,N} = L_0, L_1 \ldots L_N$.

Let us define the control of the plane motion X (1) $(w_3(t) \equiv g,\ t \geq 0)$ along the broken line $L_{0,N}$, so that the center of the illumination square moves along the segment $L_{k-1}L_k$ in a time optimal way. The controls w_1, w_2, which ensure the center of the square moves from one vertex $L_{k-1}\left(x_1^{(k-1)}, x_2^{(k-1)}\right)$ with zero speed $\dot{x}_1^{(k-1)} = \dot{x}_2^{(k-1)} = 0$ to the next vertex $L_k\left(x_1^{(k)}, x_2^{(k)}\right)$ with zero speed $\dot{x}_1^{(k)} = \dot{x}_2^{(k)} = 0$ along straight segments $L_{k-1}L_k$, are determined from the solution of a two-point optimal speed problem [9]: on vertical sections $L_{k-1}L_k$

$$\begin{aligned}
&w_1^*(t) = 0, \quad w_2^*(t) = W \sin n\{(t'/2 - t)\Delta x_2\}, \quad t_{k-1} \leq t \leq t_k, \\
&t' = 2\left(|\Delta x_2| W^{-1}\right)^{1/2}, \ t_k = t_{k-1} + t', \ k = 2n+1, \ n = 0, 1, \ldots, (N-1)/2, \\
&\Delta x_2 = x_2^{(k)} - x_2^{(k-1)} > 0, \ x_2^{(k)} = a_2 - l_0, \ x_2^{(k-1)} = l_0, \\
&k = 4p+1, \quad p = 0, 1, \ldots, P \leq (N-1)/2, \\
&\Delta x_2 = x_2^{(k-1)} - x_2^{(k)} < 0, \ x_2^{(k)} = l_0, \ x_2^{(k-1)} = a_2 - l_0, \\
&k = 4q+3, \quad q = 0, 1, \ldots, Q \leq (N-1)/2, \\
&t_0 = 0, \quad P, Q - \text{integer numbers}, \quad N - \text{odd integer};
\end{aligned} \tag{11}$$

on horizontal sections $L_{k-1}L_k$

$$\begin{aligned}
&w_1^*(t) = W \sin n\{(t''/2 - t)\Delta x_1\}, \quad w_2^*(t) = 0, \quad t_{k-1} \leq t \leq t_k, \\
&t'' = 2\left(\Delta x_1 W^{-1}\right)^{1/2}, \ t_k = t_{k-1} + t'', \ k = 2n, \ n = 1, \ldots, (N-1)/2, \\
&\Delta x_1 = x_1^{(k)} - x_1^{(k-1)} = h, \quad k = 2n, \quad n = 1, \ldots, (N-3)/2, \\
&\Delta x_1 = x_1^{(k)} - x_2^{(k-1)} \leq h, \quad k = N-1, \quad N - \text{odd integer}.
\end{aligned} \tag{12}$$

The corresponding law of variation of electric current in the circuit of the light source $j = j(t)$ is determined according to the relation (10):

$$j(t) \equiv \sqrt{E\eta\xi^{-1}R^{-1}} x_3^0, \quad t \in [0, T]. \tag{13}$$

Moving along the broken line $L_{0,N}$ the center O of the square K with the side of constant length $2l_0$ scans a rectangle in the direction of increasing x_1 in increments of h, $0 < h < 2l_0$, leaving strips with a width of l_0 on each side (top and bottom) of the rectangle.

As follows from (11), (12), the optimal travel times are the same for each vertical section and for each horizontal section of length h and are calculated, respectively, as follows:

$$t' = 2\sqrt{(a_2 - 2l_0)W^{-1}}, \quad t'' = 2\sqrt{hW^{-1}}. \tag{14}$$

Guaranteed Search with Minimal Light Energy Consumption. With the search method described in the previous section, if the following condition is satisfied

$$2t' = 4\sqrt{(a_2 - 2l_0)W^{-1}} < 2l_0V^{-1},$$

i.e. l_0 satisfies the constraint

$$l_{\min} = -4V^2W^{-1} + \left(16V^4W^{-2} + 4V^2W^{-1}a_2\right)^{1/2} < l_0 \leq Ca_3, \tag{15}$$

then selecting the scan step h from the condition

$$2t' + t'' = 4\sqrt{(a_2 - 2l_0)} + 2\sqrt{hW^{-1}} \leq (2l_0 - h)V^{-1}, \ \ h < 2l_0,$$

or, which is the same, from the segment

$$0 < h \le h^{\max},$$
$$h_{\max} = \left(-VW^{-1/2} + \left(V^2W^{-1} + 2l_0 - 4VW^{-1/2}(a_2 - 2l_0)^{1/2}\right)^{1/2}\right)^2 < 2l_0, \qquad (16)$$

the object X is guaranteed to detect the sought object Y no later than the time T.

Consider the following positive function N_1 of l_0 and h:

$$N_1(l_0, h) = (a_1 - 2l_0)h^{-1},$$
$$l_{\min} < l_0 \le Ca_3, \quad 0 < h \le h^{\max} < 2l_0. \qquad (17)$$

Function (17) is monotonically decreasing with respect to h. Consequently,

$$\min_{0<h\le h_{\max}} N_1(l_0, h) = (a_1 - 2l_0)h_{\max}^{-1} = N_1(l_0). \qquad (18)$$

We denote

$$R_0 = \{l_0 \in (l_{\min}, Ca_3]: \; N_1(l_0) = [N_1(l_0)]\}, \qquad (19)$$

where the symbol $[\cdot]$ means the integer part of a real number.

For values $l_0 \in R_0$, integer $1 + N_1(l_0)$ determines the number of vertical displacements with a scanning step $h_{\max}$ (16). Meanwhile, the moving of the center of a square along broken line $L_{0,N}$, $N = 2N_1(l_0) + 1$, ends at point $L_N = (a_1 - l_0, l_0)$, if $N_1(l_0)$ is an odd integer, and at point $L_N = (a_1 - l_0, a_2 - l_0)$, if $N_1(l_0)$ is an even integer.

With this in mind and using (14), (18), the functional (8) on the set (19) can be represented as

$$J(l_0) = \left(E\eta\xi^{-1}C^{-2}L(l_0)T(l_0)\right), \quad l_0 \in R_0,$$
$$L(l_0) = l_0^2, \;\; T(l_0) = t'(l_0) + (t'(l_0) + t''(l_0))N_1(l_0), \qquad (20)$$

where $T(l_0)$ is the guaranteed search time.

Thus, the Problem is reduced to finding the parameter $l_0^* \in R_0$ that delivers the minimum in the problem

$$\overline{J}^* = \overline{J}(l_0^*) = \min_{l_0 \in R_0} L(l_0) \cdot T(l_0). \qquad (21)$$

The functions $L(l_0)$ and $T(l_0)$ on the set R_0 take, respectively, monotonically increasing and monotonically decreasing discrete values. The minimum in (21), depending on the relations between the parameters, is reached both at the inner and at the extreme points of the set (19). Taking this into account, numerical calculations of the determination l_0^* were carried out for various values of the parameters of the Problem. In particular, for parameters $a_1 = 200\ m$, $a_2 = 100\ m$, $a_3 = 20\ m$, $C = 1$, $W = 4\ m/s^2$, $V = 0.5\ m/s$, minimum value $\overline{J}^* = 29087\ m^2\ s$ reached at the inner point $R_0 : L_0^* = 9.01\ m$. The corresponding number of complete motion cycles with a scan step $h_{\max} = 7.9\ m$ (16) equals $N_1 = 24$ (18), and guaranteed search time is $T = 346.16\ s$ (20). The electric current change in the light source circuit and the minimum amount of energy consumed by the light source are determined from (13) and (20) for specific values of parameters R, E, η, ξ of search system.

Conclusion. A simple way is proposed to control the movement of a dynamic object in the task of searching for a moving object in a rectangular area using a square

area of constant size and a given illumination. A condition is obtained that ensure successful completion of the search. An algorithm is proposed for finding the optimal size of the irradiance square, at which the sought-for object is detected during with minimal light energy consumption calculated for the guaranteed search time.

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ՇԱՐԺԱԿԱՆ ՕԲՅԵԿՏԻ ԴԻՆԱՄԻԿ ՓՆՏՐՄԱՆ ՕՊՏԻՄԱԼ ՂԵԿԱՎԱՐՈՒՄԸ ՈՒՂՂԱՆԿՅՈՒՆ ՏԻՐՈՒՅԹՈՒՄ

Դիտարկվում է դինամիկ օբյեկտի տարածական շարժումների օպտիմալ ղեկավարման խնդիրը հարթության ուղղանկյուն տիրույթում պարզ շարժումներ կատարող օբյեկտի փնտրման նպատակով: Առաջարկվել է փնտրման ղեկավարման եղանակ և լուսային աղբյուրի շղթայում էլեկտրական հոսանքի փոփոխման համապատասխան օրենք, որոնց դեպքում որոնելի օբյեկտի հայտնաբերումն իրականացվում է փնտրման երաշխավորված ժամանակում նվազագույն լուսային էներգածախսով: