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Mechanics

SHEAR WAVES IN LONGITUDINAL PERIODICAL WEAK-INHOMOGENEOUS LAYER

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In the paper characters of propagation of elastic waves in layer with continuously changing periodical heterogeneity along layer are investigated. Numerical results of dependence of wave propagation phase velocity from determining parameters of heterogeneity of wavelength are shown.

***Keywords*:** propagation of elastic waves, spread waves, weak-inhomogeneous elastic surrounding.

1. Introduction. There are numerous researches for the propagation of spread waves in flat layer at available heterogeneity in thick layer. Discussions of works in waveguide with periodical repetitive piecewise permanent heterogeneity along layer are contained [1–2]. Surface waves in weak-inhomogeneous elastic surroundings are considered in [3–5].

It is suggested that elastic waveguide takes area $-\infty < x < \infty$, $0 \le y \le b$, $-\infty < z < \infty$ in rectangular cartesian system (x, y, z). Propagation of pure elastic wave is observed (the antiplane problem)

$$u = v = 0, \qquad w = w(x, y, t) \tag{1.1}$$

with following motion equation:

$$\partial\sigma_{zx}/\partial x + \partial\sigma_{zy}/\partial y = \rho(x)\partial^2 w/\partial t^2 . \tag{1.2}$$

According to Hooke's law the density of material, as well as the shear modulus $\mu(x)$, are functions of longitudinal coordinate x:

$$\sigma_{zx} = \mu(x)\partial w/\partial x, \quad \sigma_{zy} = \mu(x)\partial w/\partial y. \tag{1.3}$$

It is accepted that $\rho(x)$, $\mu(x)$ are weak changing periodical functions

$$\mu(x) = \mu_0\left(1 + \varepsilon_1 \sin\lambda_1 x\right), \quad \rho(x) = \rho_0\left(1 + \varepsilon_2 \sin\lambda_1 x\right), \tag{1.4}$$

where $\lambda_1 = \pi/a$, $\varepsilon_1^2 << 1$, $\varepsilon_2^2 << 1$. From (1.2–1.4) the following equation is received

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$$\left(1+\varepsilon_1 \sin\lambda_1 x\right)\Delta w+\lambda_1\varepsilon_1\cos\lambda_1 x\,\partial w/\partial x=c_t^{-2}\left(1+\varepsilon_2\sin\lambda_1 x\right)\partial^2 w/\partial t^2, \quad (1.5)$$

where $\Delta=\partial^2/\partial x^2+\partial^2/\partial y^2$, $c_t^2=\mu_0/\rho_0$.

2. It is assumed that plane borders of waveguide are fixed

$$w=0 \quad \text{when} \quad y=0,\, b. \quad (2.1)$$

In that case the solution of (1.5) satisfying boundary condition (2.1) is shown with the following

$$w=w_n(x)e^{i\omega_n t}\sin\mu_n y, \quad \mu_n=\pi n/b,\ n=1,2,... \quad (2.2)$$

The substitution of (2.2) in (1.5) leads to sequential ordinary differential equations

$$\left(1+\varepsilon_1\sin\lambda_1 x\right)w_n''+\lambda_1\varepsilon_1\cos\lambda_1 x\, w_n'+\mu_n^2\left[\eta_n^2-1+\left(\varepsilon_2\eta_n^2-\varepsilon_1\right)\sin\lambda_1 x\right]w_n=0, \quad (2.3)$$

where $\eta_n^2=c_t^{-2}\mu_n^{-2}\omega_n^2$.

The solution of (2.3) with changing periodical coefficients is being looked for in the following form

$$w_n=a_{0n}+\sum_{m=1}^{\infty}(a_{mn}\cos\lambda_m x+b_{mn}\sin\lambda_m x),\ \ \lambda_m=\pi m/a. \quad (2.4)$$

Demanding (2.4) to satisfy (2.3) is received sequential related system of homogeneous algebraic equations:

$$\mu_n^2(\eta_n^2-1)a_{0n}+(\beta_1+\lambda_1^2\varepsilon_1)b_{1n}/2=0, \quad (2.5)$$

$$\begin{cases}\mu_n^2(\varepsilon_2\eta_n^2-\varepsilon_1)a_{0n}+\alpha_1 b_{1n}-(\beta_2+\lambda_1\lambda_2\varepsilon_1)a_{2n}/2=0,\\ \alpha_1 a_{1n}+(\beta_2+\lambda_1\lambda_2\varepsilon_1)b_{2n}/2=0\end{cases} \quad (2.6)$$

and for $m\ge 2$

$$\begin{cases}(\beta_{m-1}-\lambda_1\lambda_{m-1}\varepsilon_1)a_{m-1n}/2+\alpha_m b_{mn}-(\beta_{m+1}+\lambda_1\lambda_{m+1}\varepsilon_1)a_{m+1n}/2=0,\\ -(\beta_{m-1}-\lambda_1\lambda_{m-1}\varepsilon_1)b_{m-1n}/2+\alpha_m a_{mn}+(\beta_{m+1}+\lambda_1\lambda_{m+1}\varepsilon_1)b_{m+1n}/2=0.\end{cases} \quad (2.7)$$

In (2.5)–(2.7) are accepted the following designations

$$\alpha_m=\mu_n^2(\eta_n^2-1)-\lambda_m^2, \quad \beta_m=\mu_n^2(\varepsilon_2\eta_n^2-\varepsilon_1)-\varepsilon_1\lambda_m^2. \quad (2.8)$$

3. Zero-order approximation according to (2.5)

$$\mu_n^2(\eta_n^2-1)\,a_{0n}=0,$$

so

$$\eta_n^2=1. \quad (3.1)$$

In this approximation x (thick variations) undependent frequencies of variations are received. For first approximation from (2.5) and (2.6), taking into account conditions $a_{2n}=0$, $b_{2n}=0$ and designation (2.8), will obtain

$$\begin{cases}\mu_n^2(\eta_n^2-1)\,a_{0n}+(\beta_1+\lambda_1^2\varepsilon_1)\,b_{1n}/2=0,\\ \mu_n^2(\varepsilon_2\eta_n^2-\varepsilon_1)\,a_{0n}+[\mu_n^2(\eta_n^2-1)-\lambda_1^2]b_{1n}=0,\\ [\mu_n^2(\eta_n^2-1)-\lambda_1^2]a_{1n}=0.\end{cases} \quad (3.2)$$

The equality of determinant of system (3.2) to zero leads to two equations with η_n^2:

$$\eta_n^2-1-\frac{\lambda_1^2}{\mu_n^2}=0 \text{ and } \left(1-\frac{\varepsilon_2^2}{2}\right)\eta_n^4-2\left(1+\frac{\lambda_1^2}{2\mu_n^2}-\frac{\varepsilon_1\varepsilon_2}{2}\right)\eta_n^2+\left(1+\frac{\lambda_1^2}{\mu_n^2}-\frac{\varepsilon_1^2}{2}\right)=0. \quad (3.3)$$

First equation will have the following solution

$$\eta_n^2=1+\frac{\lambda_1^2}{\mu_n^2} \text{ or } \eta_n^2=1+\left(\frac{b}{a}n\right)^2. \quad (3.4)$$

The solution of second (biquadratic) equation will have the following form

$$\left(1-\frac{\varepsilon_2^2}{2}\right)\eta_n^2=1+\frac{\lambda_1^2}{2\mu_n^2}-\frac{\varepsilon_1\varepsilon_2}{2}\pm\sqrt{\left(1+\frac{\lambda_1^2}{2\mu_n^2}-\frac{\varepsilon_1\varepsilon_2}{2}\right)^2-\left(1+\frac{\lambda_1^2}{\mu_n^2}-\frac{\varepsilon_1^2}{2}\right)\left(1-\frac{\varepsilon_2^2}{2}\right)}. \quad (3.5)$$

Here the signs – and + match with thick variation and with the appearance of new phasic velocities correspondingly.

4. Let introduce new variables $\xi_n=\mu_n^2/\lambda_1^2=(an/b)^2$, $\Omega_n^2=\omega_n^2/c_t^2\lambda_1^2=\eta_n^2\xi_n$. With new variables relations (2.5)–(2.7) can be shown in the following forms

$$b_{1n}=-\frac{2A_0}{D_1}a_{0n}, \quad (4.1)$$

$$\begin{cases} a_{2n}=\dfrac{2D_0}{D_2}a_{0n}+\dfrac{2A_1}{D_2}b_{1n}, \\ b_{2n}=-\dfrac{2A_1}{D_2}a_{1n}, \end{cases} \quad (4.2)$$

$$\begin{cases} a_{m+1\,n}=\dfrac{D_m}{D_{m+1}}a_{m-1\,n}+\dfrac{2A_m}{D_{m+1}}b_{mn}, \\ b_{m+1\,n}=\dfrac{D_m}{D_{m+1}}b_{m-1\,n}-\dfrac{2A_m}{D_{m+1}}a_{mn}=0, \quad m=2,3,..., \end{cases} \quad (4.3)$$

where

$$A_k=\Omega_n^2-\xi_n-k^2,\ D_k=\varepsilon_2\Omega_n^2-\varepsilon_1\xi_n-\varepsilon_1 k(k-1),\ k=0,1,2,... \quad (4.4)$$

Taking into account (4.4) for $b_{1n}=0$, from (4.1) will have the following

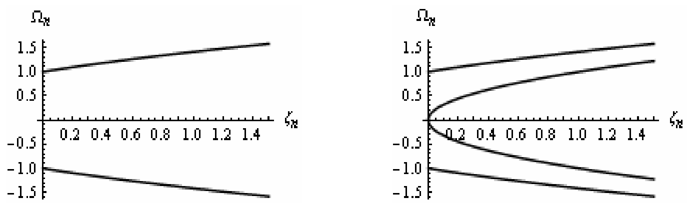
$$\frac{2(\Omega_n^2-\xi_n)}{\varepsilon_2\Omega_n^2-\varepsilon_1\xi_n}a_{0n}=0. \quad (4.5)$$

We will obtain $\Omega_n^2-\xi_n=0$, because of condition of nontrivial solution and $a_{0n}\neq 0$. This is an equation for zero approximation, but only for new variables. From (4.1) and (4.2), for $a_{0n}\neq 0$, $a_{1n}\neq 0$ (non triviality) and $a_{2n}=0$, $b_{2n}=0$ we will obtain two equations for first approximation:

$$2D_0/D_2-4A_0A_1/D_1D_2=0, \quad -2A_1/D_2=0. \quad (4.6)$$

These two equations coincide with equations written in (3.3). In common case, applying difference equations (4.1)–(4.3), we can calculate coefficients $a_{m+1\,n}$ and $b_{m+1\,n}$ for any $m=1,2,3,...$ Then each of $a_{m+1\,n}$ and $b_{m+1\,n}$ will have common multiplier a_{0n} or a_{1n}. We will obtain two equations for m-order approximation interrupting calculations with substitutions $a_{m+1\,n} = 0$ and $b_{m+1\,n} = 0$, and by condition of nontrivial solution, i.e. $a_{0n} \neq 0$, $a_{1n} \neq 0$. These equations coincide with the equations received from determinant that has the form of two multipliers.

In the case of first order approximation the solution of one from two equations gives the curves shown in Fig. 1,a and the solution curves of the other equation are shown in Fig. 1,b. Here and in further cases the calculations are made for $\varepsilon_1 = 0.1$, $\varepsilon_2 = 0.05$.



a b

Fig. 1.

Solution curves: a) for one equation from two in first order approximation;
b) for the other equation in first order approximation.

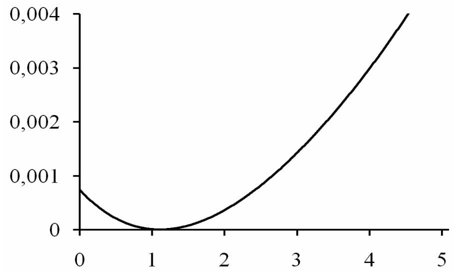
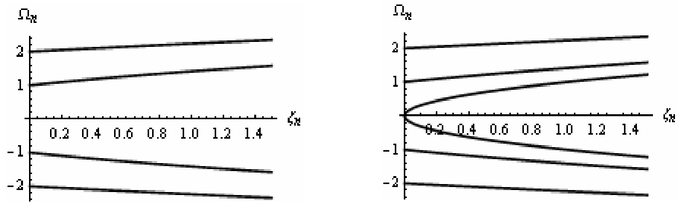


Fig. 2. Difference of nearly overlayed curves.

Overlaying these two graphs will obtain graph similar to Fig. 1,b, but we can not state they overlay. Fig. 2 shows the difference of nearly overlayed curves. This difference becomes more for bigger ξ_n. In fact we have 6 curves in the case of first order approximation.

In the case of second order approximation the solution curves for two equations are shown in Fig. 3,a and Fig. 3,b separately.



a b

Fig. 3.

Solution curves: a) for one equation from two in second order approximation;
b) for the other equation in second order approximation.

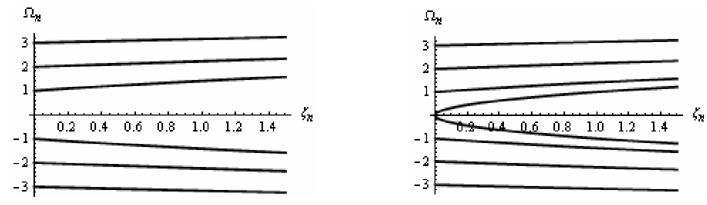


Fig. 4.

Solution curves: a) for one equation from two in third order approximation;
b) for the other equation in third order approximation.

In Fig. 3,a is given the graph of more exact frequency Fig. 1,a and two new curves of frequency. Analogous graphs in third order approximation are shown in Fig. 4,a and Fig. 4,b.

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