

MEAN DISTANCE BETWEEN TWO POINTS IN A DOMAIN

N. G. AHARONYAN*

Chair of Probability Theory and Mathematical Statistics YSU, Armenia

Let D be a bounded convex domain in the Euclidean plane and we choose uniformly and independently two points in D . How large is the mean distance $m(D)$ between these two points? Up to now, there were known explicit expressions for $m(D)$ only in three cases, when D is a disc, an equilateral triangle and a rectangle. In the present paper a formula for calculation of mean distance $m(D)$ by means of the chord length density function of D is obtained. This formula allows to find $m(D)$ for those domains D , for which the chord length distribution is known. In particular, using this formula, we derive explicit forms of $m(D)$ for a disc, a regular triangle, a rectangle, a regular hexagon and a rhombus.

Keywords: chord length distribution function, mean distance, convex domain geometry.

Introduction. Let D be a bounded convex domain in the Euclidean plane \mathbb{R}^2 (with area $\|D\|$ and perimeter $/D/$), and we choose uniformly and independently two points P_1 and P_2 from D . How large is the mean distance $m(D)$ between these two points? In other words, we want to calculate the following quantity [1]:

$$m(D) = 1/\|D\|^2 \int \int_{D,D} \rho(P_1, P_2) dP_1 dP_2, \tag{1.1}$$

where $\rho(P_1, P_2)$ is the Euclidean distance between P_1 and P_2 , dP_i ($i=1,2$) is the two dimensional Lebesgue measure. Up to now, there were known explicit expressions for $m(D)$ only in three cases when D is the disc K_r of a radius r [2]:

$$m(K_r) = 128r/45\pi \tag{1.2}$$

for an equilateral triangle Δ_a of side a :

$$m(\Delta_a) = a(1/5 + 3\ln/20), \tag{1.3}$$

and for a rectangle $R_{a,b}$ with sides $a \leq b$ [3]:

$$m(R_{a,b}) = \frac{1}{15} \left[\frac{a^3}{b^2} + \frac{b^3}{a^2} + \sqrt{a^2 + b^2} \left(3 - \frac{a^2}{b^2} - \frac{b^2}{a^2} \right) + \frac{5}{2} \left(\frac{b^2}{a} \ln \frac{a + \sqrt{a^2 + b^2}}{b} + \frac{a^2}{b} \ln \frac{b + \sqrt{a^2 + b^2}}{a} \right) \right]. \tag{1.4}$$

* E-mail: narine78@ysu.am

In the present paper a formula for calculation of mean distance $m(D)$ by means of the chord length density function of D is obtained, which allows to find an explicit form of mean distance $m(D)$ for those domains D , for which the chord length distribution is known. In particular, using this formula, we derive explicit forms of $m(D)$ for a disc, a regular triangle, a rectangle, a regular hexagon and a rhombus.

2. The Main Formula. In this section we obtain a formula for calculation $m(D)$ in terms of chord length density function for domain D .

Let $\chi(g) = g \cap D$, $g \in [D]$, is a chord in D , where $[D] = \{g \in \mathbb{G} : g \cap D \neq \emptyset\}$ is the set of lines intersecting domain D (\mathbb{G} is the space of all undirected lines in the plane \mathbb{R}^2). It is well known the following formula between $m(D)$ and the integral

$$I_4 = \int_{[D]} \chi^4(g) dg :$$

$$I_4 = 6 \|D\|^2 m(D), \quad (2.1)$$

where dg is the locally finite measure in the space \mathbb{G} , which is invariant with respect to the group of all Euclidean motions in the plane [1, 4].

By definition, $F_D(y) = 1/|\partial D| \int_{\chi(g) \leq y} dg$ is the chord length distribution function for D [5, 6].

To transform the formula for $m(D)$, we write

$$I_4 = |\partial D| \int_0^{+\infty} x^4 f_D(x) dx, \quad (2.2)$$

where $f_D(x)$ is the chord length density function of D (that is $f_D(x) = F_D'(x)$ is the first derivative of the distribution function). Finally, from (2.2) and (2.1), we get

$$m(D) = |\partial D| / 6 \|D\|^2 \int_0^{+\infty} x^4 f_D(x) dx, \quad (2.3)$$

Therefore, if we know the explicit form of $f_D(x)$ for domain D , using (2.3) we can calculate $m(D)$. Further we demonstrate the efficiency of formula (2.3).

3. The Case of a Disc. Consider a disc of radius r , $D = K_r$. In this case the chord length density function has the following form [5]:

$$f_{K_r}(x) = \begin{cases} 0, & \text{if } x \notin (0, 2r), \\ x/(4r^2 \sqrt{1 - x^2/4r^2}), & \text{if } x \in (0, 2r). \end{cases}$$

Therefore, we get $m(K_r) = 1/6\pi r^4 \int_0^{2r} x^5 / \sqrt{4r^2 - x^2} dx = 128r/45\pi$, i.e. (1.2) [1, 5, 7].

4. The Case of a Regular Triangle. The chord length density function for a regular triangle Δ_a with side a has the following form [5, 8, 9]:

$$f_{\Delta_a}(x) = \begin{cases} 0, & \text{if } x \notin [0, a]; \\ \frac{1}{2a} + \frac{\pi}{3a\sqrt{3}}, & \text{if } x \in \left(0, \frac{a\sqrt{3}}{2}\right]; \\ \frac{1}{2a} - \frac{\sqrt{4x^2 - 3a^2}}{2x^2} + \frac{2}{a\sqrt{3}} \arcsin \frac{a\sqrt{3}}{2x} - \frac{2\pi}{3a\sqrt{3}}, & \text{if } x \in \left(\frac{a\sqrt{3}}{2}, a\right]. \end{cases} \quad (4.1)$$

Substituting the values of $f_{\Delta_a}(x)$ from (4.1) into (2.3), we obtain

$$m(\Delta_a) = \frac{8}{3a^3} \left[\frac{1}{2a} \int_0^a x^4 dx + \frac{\pi}{3a\sqrt{3}} \int_0^{\sqrt{3}a/2} x^4 dx - \frac{2\pi}{3a\sqrt{3}} \int_{\sqrt{3}a/2}^a x^4 dx - \frac{1}{2} \int_{\sqrt{3}a/2}^a x^2 \sqrt{4x^2 - 3a^2} dx + \frac{2}{a\sqrt{3}} \int_{\sqrt{3}a/2}^a x^4 \arcsin \frac{a\sqrt{3}}{2x} dx \right]. \quad (4.2)$$

Using some standard formulas we obtain $m(\Delta_a) = a(1/5 + 3 \ln 3/20)$ (1.3) [1–3, 5].

5. The Case of a Rectangle. It is well known that the chord length density function for a rectangle $R_{a,b}$ with sides $a \leq b$ has the following form [5–7, 9, 10]:

$$f_{R_{a,b}}(x) = \begin{cases} 0, & \text{if } x \notin [0, \sqrt{a^2 + b^2}]; \\ x/(a+b), & \text{if } x \in (0, a]; \\ a^2b/(a+b)x^2\sqrt{x^2 - a^2}, & \text{if } x \in (a, b]; \\ \frac{ab}{(a+b)x^2} \left(\frac{a}{\sqrt{x^2 - a^2}} + \frac{b}{\sqrt{x^2 - b^2}} \right) - \frac{1}{a+b}, & \text{if } x \in (b, \sqrt{a^2 + b^2}]. \end{cases} \quad (5.1)$$

Substituting the values of $f_{R_{a,b}}(x)$ from (5.1) into (2.3), we obtain

$$m(R_{a,b}) = \frac{1}{3a^2b^2} \left[\frac{a^5}{5} + a^2b \int_a^{\sqrt{a^2+b^2}} \frac{x^2}{\sqrt{x^2 - a^2}} dx + ab^2 \int_b^{\sqrt{a^2+b^2}} \frac{x^2}{\sqrt{x^2 - b^2}} dx + \frac{b^5}{5} - \frac{(a^2 + b^2)^2 \sqrt{a^2 + b^2}}{5} \right]. \quad (5.2)$$

After some calculations we obtain (1.4) [1, 2]. In the case $a = b$ we get the mean distance for a square with side a

$$m(\square_a) = a/15(2 + \sqrt{2} + 5 \ln(1 + \sqrt{2})). \quad (5.3)$$

6. The Case of a Regular Hexagon. For a regular hexagon H_a with side a we have: $|H_a| = 6a$, $\|H_a\| = 3\sqrt{3}a^2/2$. The density function has the following form (see [6, 8–10]):

$$f_{H_a}(y) = \begin{cases} 0, & \text{if } y \notin (0, 2a]; \\ \frac{1}{a} \left(\frac{1}{2} - \frac{\pi}{6\sqrt{3}} \right), & \text{if } y \in (0, a]; \\ \frac{\pi}{2a\sqrt{3}} - \frac{2}{a\sqrt{3}} \arcsin \frac{a\sqrt{3}}{2y} + \frac{\sqrt{4y^2 - 3a^2}}{2y^2}, & \text{if } y \in (a, a\sqrt{3}]; \\ \frac{\pi}{6a\sqrt{3}} - \frac{1}{2a} - \frac{1}{a\sqrt{3}} \arccos \frac{a\sqrt{3}}{y} + \frac{6a^2 - y^2}{y^2 \sqrt{y^2 - 3a^2}}, & \text{if } y \in (a\sqrt{3}, 2a]. \end{cases} \quad (6.1)$$

Therefore, $m(H_a) = \frac{4}{27a^3} \int_0^\infty x^4 f_{H_a}(x) dx$. Substituting (6.1) form, we get

$$m(H_a) = \frac{4}{27a^3} [I_1 + I_2 + I_3], \text{ where } I_1 = \frac{1}{a} \int_0^a \left(\frac{1}{2} - \frac{\pi}{6\sqrt{3}} \right) x^4 dx = a^4 \left(\frac{1}{10} - \frac{\pi}{30\sqrt{3}} \right);$$

$$I_2 = \frac{\pi}{2a\sqrt{3}} \int_a^{a\sqrt{3}} x^4 dx - \frac{2}{a\sqrt{3}} \int_a^{a\sqrt{3}} x^4 \arcsin \frac{a\sqrt{3}}{2x} dx + \int_a^{a\sqrt{3}} \frac{x^2 \sqrt{4x^2 - 3a^2}}{2} dx;$$

$$I_3 = \int_{a\sqrt{3}}^{2a} \left(\frac{\pi}{6a\sqrt{3}} - \frac{1}{2a} \right) x^4 dx - \frac{1}{a\sqrt{3}} \int_{a\sqrt{3}}^{2a} x^4 \arccos \frac{a\sqrt{3}}{x} dx + \int_{a\sqrt{3}}^{2a} \frac{6a^2 - x^2}{a\sqrt{3} \sqrt{x^2 - 3a^2}} x^2 dx.$$

Calculating I_2 and I_3 , we obtain finally:

$$m(H_a) = a \left(\frac{4\pi}{15} - \frac{227}{2160} + \frac{478\pi}{3645\sqrt{3}} + \frac{113\sqrt{3}}{240} - \frac{3}{160} \ln \frac{2+\sqrt{3}}{\sqrt{3}} + \frac{14}{15} \ln \sqrt{3} \right).$$

7. The Case of Rhombus. In terms of elementary functions the chord length distribution function for rhombus $R(a, \gamma)$ for $\pi/3 < \gamma \leq \pi/2$ has the form [10]:

$$f_{R(a, \gamma)}(x) = \begin{cases} 0, & \text{if } x \notin [0, 2a \cos \gamma/2]; & (1 + (\pi/2 - \gamma) \cot \gamma) / 2a, & \text{if } x \in [0, a \sin \gamma]; \\ \frac{1}{2a} \left(1 - \left(\frac{\pi}{2} + \gamma - 2 \arcsin \frac{a \sin \gamma}{x} \right) \cot \gamma \right) + \frac{a^2 \sin^2 \gamma + x^2 \cos \gamma}{x^2 \sqrt{x^2 - a^2 \sin^2 \gamma}}, & \text{if } x \in [a \sin \gamma, a]; \\ -\frac{1}{2a} \left(1 + \left(\frac{\pi}{2} - \gamma \right) \cot \gamma \right) + \frac{a^2 \sin^2 \gamma}{x^2 \sqrt{x^2 - a^2 \sin^2 \gamma}}, & \text{if } x \in \left[a, 2a \sin \frac{\gamma}{2} \right]; \\ \frac{-1 + \left(\gamma - 2 \arcsin \frac{a \sin \gamma}{x} \right) \cot \gamma}{4a} + \frac{a^2 \sin^2 \gamma + x^2 \cos \gamma}{2x^2 \sqrt{x^2 - a^2 \sin^2 \gamma}}, & \text{if } x \in \left[2a \sin \frac{\gamma}{2}, 2a \cos \frac{\gamma}{2} \right]. \end{cases}$$

Since $|R(a, \gamma)| = 4a$, $\|R(a, \gamma)\| = a^2 \sin \gamma$, using (2.3) we obtain

$$m(R(a, \gamma)) = \frac{2}{3a^3 \sin^2 \gamma} \int_0^\infty x^4 f_{R(a, \gamma)}(x) dx.$$

We denote by $m_1(R(a, \gamma)) = \int_0^{a \sin \gamma} \frac{x^4}{2a} \left[1 + \left(\frac{\pi}{2} - \gamma \right) \cot \gamma \right] dx$,

$$m_2(R(a, \gamma)) = \int_{a \sin \gamma}^a \frac{x^4}{2a} \left[\left(1 - \left(\frac{\pi}{2} + \gamma - 2 \arcsin \frac{a \sin \gamma}{x} \right) \cot \gamma \right) \right] dx$$

and $m_3(R(a, \gamma)) = \int_{a \sin \gamma}^a \frac{a^2 x^2 \sin^2 \gamma + x^4 \cos \gamma}{x^2 \sqrt{x^2 - a^2 \sin^2 \gamma}} dx$.

We have $m_2(R(a, \gamma)) = \frac{a^4}{10} - \frac{a^4 \sin^5 \gamma}{10} - \frac{a^4 (\pi + 2\gamma) \cot \gamma}{20} + \frac{a^4 (\pi + 2\gamma) \cos \gamma \sin^4 \gamma}{20} + \frac{\cot \gamma}{a} G_1$,

where $G_1 = a^5 \sin^5 \gamma \int_{\sin \gamma}^1 (1/x^6) \arcsin x dx$. Calculating G_1 and $m^3(R(a, \gamma))$, we get

$$G_1 = -a^5 \sin^5 \gamma \left[\frac{\pi}{10} - \frac{\gamma}{5 \sin^5 \gamma} - \frac{1}{5} \left[\frac{\cos \gamma}{4 \sin^4 \gamma} + \frac{3 \cos \gamma}{8 \sin^2 \gamma} + \frac{3}{8} \ln \frac{1 + \cos \gamma}{\sin \gamma} \right] \right] \text{ and}$$

$$m_3(R(a, \gamma)) = a^4 \left[\frac{\sin^2 \gamma \cos \gamma}{2} + \frac{\sin^4 \gamma}{2} \left(1 + \frac{3}{4} \cos \gamma \right) \ln \frac{1 + \cos \gamma}{\sin \gamma} + \frac{\cos^2 \gamma}{4} + \frac{3}{8} \sin^2 \gamma \cos^2 \gamma \right].$$

Further, we have

$$m_4(R(a, \gamma)) = \left[\frac{a^4}{10} - \frac{16a^4 \sin^5(\gamma/2)}{5} \right] \left[1 + \left(\frac{\pi}{2} - \gamma \right) \cot \gamma \right] + M_1 a^2 \sin^2 \gamma$$

$$m_5(R(a, \gamma)) = \frac{8}{5} \left[a^4 \sin^5(\gamma/2) - a^4 \cos^5(\gamma/2) + \gamma a^4 \cot \gamma \cos^5(\gamma/2) - \gamma a^4 \cot \gamma \sin^5(\gamma/2) \right] - K_1 \cot \gamma / 2a,$$

$$\text{where } M_1 = \int_a^{2a \sin(\gamma/2)} \frac{x^2}{\sqrt{x^2 - a^2 \sin^2 \gamma}} dx \quad \text{and} \quad K_1 = \int_{2a \sin(\gamma/2)}^{2a \cos(\gamma/2)} x^4 \arcsin \frac{a \sin \gamma}{x} dx,$$

$$\text{and at least } m_6(R(a, \gamma)) = \frac{1}{2} \int_{2a \sin(\gamma/2)}^{2a \cos(\gamma/2)} \frac{a^2 x^2 \sin^2 \gamma + x^4 \cos \gamma}{\sqrt{x^2 - a^2 \sin^2 \gamma}} dx.$$

Using a standard integral, we obtain

$$M_1 = a^2 \left(2 \sin^3 \frac{\gamma}{2} - \frac{\cos \gamma}{2} \right) + \frac{a^2}{2} \sin^2 \gamma \ln \frac{2 \sin \frac{\gamma}{2} (1 + \sin \frac{\gamma}{2})}{1 + \cos \gamma}, \text{ besides we get}$$

$$K_1 = -a^5 \sin^5 \gamma \left[-\frac{\gamma}{10 \sin^5(\gamma/2)} + \frac{\pi - \gamma}{10 \cos^5(\gamma/2)} - \frac{\cos(\gamma/2)}{20 \sin^4(\gamma/2)} + \frac{\sin(\gamma/2)}{20 \cos^4(\gamma/2)} - \frac{3 \cos(\gamma/2)}{40 \sin^2(\gamma/2)} + \frac{3 \sin(\gamma/2)}{40 \cos^2(\gamma/2)} - \frac{3}{40} \ln \frac{\cos(\gamma/2) + \cos^2(\gamma/2)}{\sin(\gamma/2) + \sin^2(\gamma/2)} \right].$$

Since $m(R(a, \gamma)) = \frac{2}{3a^3 \sin^2 \gamma} \sum_{i=1}^6 m_i(R(a, \gamma))$, finally we have

$$\begin{aligned} m(R(a, \gamma)) &= \frac{2a}{3 \sin^2 \gamma} \left[\frac{1}{5} + \frac{\sin \gamma \cos \gamma}{20} + \frac{3}{40} \sin^3 \gamma \cos \gamma + \frac{1}{20} \sin^4 \gamma (10 + 9 \cos \gamma) \ln \frac{1 + \cos \gamma}{\sin \gamma} + \frac{\cos^2 \gamma}{4} + \right. \\ &+ \frac{3}{8} \sin^2 \gamma \cos^2 \gamma - \frac{8}{5} \sin^5 \frac{\gamma}{2} + \frac{8\pi}{5} \sin^5 \frac{\gamma}{2} \cot \gamma + \sin^2 \gamma \sin^3 \frac{\gamma}{2} + \frac{\sin^4 \gamma}{2} \ln \frac{2 \sin \frac{\gamma}{2} (1 + \sin \frac{\gamma}{2})}{1 + \cos \gamma} - \\ &- \frac{8}{5} \cos^5 \frac{\gamma}{2} + \frac{8}{5} \gamma \cot \gamma \left(\cos^5 \frac{\gamma}{2} - \sin^5 \frac{\gamma}{2} \right) + \frac{\sin^4 \gamma \cos \gamma}{2} \left[-\frac{\gamma}{10 \sin^5(\gamma/2)} + \frac{\pi - \gamma}{10 \cos^5(\gamma/2)} - \right. \\ &- \left. \frac{\cos \frac{\gamma}{2}}{20 \sin^4 \frac{\gamma}{2}} + \frac{\sin \frac{\gamma}{2}}{20 \cos^4 \frac{\gamma}{2}} - \frac{3 \cos \frac{\gamma}{2}}{40 \sin^2 \frac{\gamma}{2}} + \frac{3 \sin \frac{\gamma}{2}}{40 \cos^2 \frac{\gamma}{2}} \right] + \sin^2 \gamma \cos^3 \frac{\gamma}{2} + 2 \cos \gamma \left(\cos^5 \frac{\gamma}{2} - \sin^5 \frac{\gamma}{2} \right) + \\ &+ \left. \frac{3}{4} \sin^2 \gamma \cos^3 \frac{\gamma}{2} \cos \gamma - \frac{3}{4} \sin^2 \gamma \sin^3 \frac{\gamma}{2} \cos \gamma + \frac{1}{80} \sin^4 \gamma (20 + 3 \cos \gamma) \ln \frac{\cos \frac{\gamma}{2} + \cos^2 \frac{\gamma}{2}}{\sin \frac{\gamma}{2} + \sin^2 \frac{\gamma}{2}} \right]. \end{aligned}$$

Therefore, $m(R(a, \gamma)) = C(\gamma)a$, where $C(\gamma)$ is a constant depending only on the angel $\gamma \in [\pi/3, \pi/2]$.

In the case $\gamma = \pi/2$, $C(\pi/2)$ coincides with the rectangle (5.3)

$$C(\pi/2) = 1/15 \left[2 + \sqrt{2} + 5 \ln(1 + \sqrt{2}) \right].$$

Similarly we can consider the case $\gamma \in [0, \pi/3)$.

8. Conclusion. We have calculated explicit values for $m(D)$ for some particular cases, for which there exist explicit form of the chord length density function. We would like to stress that in [8] there exist an explicit form for the chord length distribution function for any regular polygon. In particular, for a regular pentagon the explicit form for chord length distribution you can find in [5]. Therefore, using the result of [8], we can calculate $m(D)$ for any regular polygon.

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Տիրույթի երկու կետերի միջև եղած միջին հեռավորությունը

Դիցուք D -ն սահմանափակ ուռուցիկ տիրույթ է E^2 -ում: Ընտրենք անկախ և հավասարաչափ բաշխումով երկու կետեր D -ից: Ներկայումս հայտնի են այդ երկու կետերի միջև եղած $m(D)$ միջին հեռավորության բացահայտ տեսքերը միայն այն դեպքերում, երբ D -ն հանդիսանում է շրջան, հավասարակողմ եռանկյուն և ուղղանկյուն: Այս հոդվածում, D տիրույթի լարի երկարության խտության ֆունկցիայի օգնությամբ ստացվել է $m(D)$ -ն հաշվելու համար բանաձև, որի օգնությամբ հաշվել ենք $m(D)$ -ն այնպիսի D -երի համար, որոնցում լարի երկարության բաշխումը հայտնի է: Մասնավորապես, օգտագործելով այդ բանաձևը, դուրս են բերվում $m(D)$ -ի բացահայտ տեսքերը շրջանի, կանոնավոր եռանկյան, ուղղանկյան, ինչպես նաև, կանոնավոր վեցանկյան և շեղանկյան համար:

Среднее расстояние между двумя точками области

Независимо и с равномерным распределением выбраны две точки из ограниченной выпуклой области (D) в евклидовой плоскости. В настоящее время существуют явные выражения для среднего расстояния $m(D)$ между этими точками для круга, равностороннего треугольника и прямоугольника. В статье приведена формула для вычисления $m(D)$ с помощью плотности длины хорды области D . Она позволяет находить $m(D)$ для тех D , для которых распределение длины хорды известно. В частности с использованием этой формулы выведены явные выражения $m(D)$ для круга, правильного треугольника, прямоугольника, правильного шестиугольника и ромба.