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ON TERMINATION OF FUNCTIONAL SYMBOL-FREE LOGIC PROGRAMS

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The present article is devoted to the termination of logic programs, which do not use functional symbols (FSF programs). A program *P* is terminating with respect to a goal *G*, if the SLD-tree of *P* and *G* is finite. In general, FSF programs are not terminating. A transformation is introduced, by which any FSF program is transformed into another, not FSF program, which is shown to be terminating with respect to the permitted goals of the original program. The program obtained via transformation and the original program are Δ-equivalent.

Keywords: logic programming, termination, functional symbol-free logic programs, transformation.

1. Introduction. Termination of logic programs is of course of utmost importance. The question whether the top-down evaluation of a goal *G* terminates with respect to (*wrt*) a logic program *P* is actually underspecified, given the fact that this evaluation may depend on the selection of atoms from goals and on the choice of the program clauses. In this paper termination is considered in the strong sense, i.e. irrespective of the selection of atoms in the goal and of the choice of program clauses. This is the most demanding notion of termination. Less demanding approaches are:

(1) termination for a fixed selection rule and for any choice of program clauses;

(2) termination for some selection rule, depending on *P* and *G*, and for any choice of program clauses.

The approach under (1) is taken by Plumer in [1] and applies to Prolog in the case that the leftmost selection rule is adopted. The approach under (2) is taken by Ullman and van Gelder [2] and applies to a system like NAIL!

In [3] it is shown that termination issue for arbitrary logic programs is not solvable. Thus, it seems appropriate to study specific cases of logic programs.

We study termination of logic programs that do not use functional symbols of arity ≥ 1 (henceforth referred to as FSF programs).

A decidable interpreter is known to exist for FSF programs [4]. However, in general, FSF programs are not terminating.

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The technical tool we shall use is called *level mapping* by Cavedon [5], who studied various classes of logic programs with negation. Level mapping is a function assigning natural numbers to variable-free atoms.

In [6] it is shown, that if a logic program is *recurrent*, i.e. if the heads of variable-free instances of program clauses have higher levels than the atoms occurring in the body of the same instance, then it is terminating with respect to bounded goals, i.e. goals whose instances are below some fixed level.

We present a transforming algorithm, by which any FSF program is transformed into a Δ-equivalent recurrent program and show that it is terminating with respect to permitted goals.

2. Notations and Background. Consider three non-intersecting sets Φ, Π and X. Φ is a set of functional symbols each possessing an arity. For any $n \geq 0$, Φ contains a countable number of symbols of arity n. Π is a set of predicate symbols each possessing an arity. For any $n \geq 0$, Π contains a countable number of symbols of arity n. X is a countable set of variables. Terms are composed of elements of sets Φ and X. The atoms are defined as usual [7]. A formula of the first-order predicate logic with equality over logical operations $\neg, \vee, \wedge, \rightarrow, \leftrightarrow$ and quantifiers $\exists$ and $\forall$ is defined conventionally [7].

A *ground term* is a term not containing variables. Similarly, a *ground atom* is an atom not containing variables. The *Herbrand universe* U is the set of ground terms and the *Herbrand base* B is the set of ground atoms. A *Herbrand interpretation* I is a subset of the Herbrand base. The value of a closed formula F in interpretation I is defined as usual. A closed formula F is a logical consequence of interpretation I ($I \models F$), if I is a model of F. For closed formulas F and G the relation $F \models G$ means that every Herbrand model of F is a model of G too.

A *program clause* is a construct of the form $A \leftarrow B_1, \ldots, B_m$, where $m \geq 0$, $B_1, \ldots, B_m$ and A are atoms. The atom A is called the head and the sequence of atoms $B_1, \ldots, B_m$ the body of the clause. In case of $m = 0$, the clause is called a fact and is denoted as A. Intuitively, a clause $A \leftarrow B_1, \ldots, B_m$ is to be interpreted as the formula $\forall (B_1 \wedge \ldots \wedge B_m \rightarrow A)$.

A *logic program* is a finite set of program clauses.

A *goal* is a construct of the form $\leftarrow C_1, \ldots, C_k$, where $C_1, \ldots, C_k$ are atoms. A goal, in which the sequence $C_1, \ldots, C_k$ is empty, is called an *empty goal*. Intuitively, a non-empty goal $\leftarrow C_1, \ldots, C_k$ is to be interpreted as the formula $\exists (C_1 \wedge \ldots \wedge C_k)$.

A *substitution* $\theta = \{x_1/t_1, \ldots, x_n/t_n\}$ is a finite set of pairs of variables and terms, where t_i is distinct from x_i and the variables $x_1, \ldots, x_n$ are also distinct. A *simple expression* is either a term or an atom. If E is a simple expression, then the *instance* of E by θ, denoted by $E\theta$, is the simple expression obtained from E by simultaneously replacing each occurrence of the variable x_i in E by the term t_i, where $i = 1, \ldots, n$. If $E\theta$ is ground, then $E\theta$ is called a *ground instance* of E. The composition of substitutions is defined traditionally [7]. If $W = \{E_1, \ldots, E_n\}$ is a finite set of simple expressions and θ is a substitution, then $W\theta$ denotes the set $\{E_1\theta, \ldots, E_n\theta\}$. Let W be a finite set of simple expressions. A substitution θ is called a *unifier* for

W, if $W\theta$ is a singleton. A unifier θ is called a *most general unifier* (mgu) for W, if for each unifier σ of W there exists a substitution γ such that $\sigma = \theta\gamma$.

If S is a program clause of the form $A \leftarrow B_1, \dots, B_m$, $m \geq 0$, then $S\theta$ denotes the clause $A\theta \leftarrow B_1\theta, \dots, B_m\theta$. If G is a goal of the form $\leftarrow C_1, \dots, C_k$, $k > 0$, then $G\theta$ denotes the goal $\leftarrow C_1\theta, \dots, C_k\theta$.

We will denote the set of variables occurring inside a simple expression E by $Var(E)$. The set of variables of a program clause S of the form $A \leftarrow B_1, \dots, B_m$ we will denote by $Var(S)$ and define as $Var(S) = Var(A) \cup Var(B_1) \cup \dots \cup Var(B_m)$.

The set of variables of a goal G of the form $\leftarrow C_1, \dots, C_k$ we will denote by $Var(G)$ and define as $Var(G) = Var(C_1) \cup \dots \cup Var(C_k)$.

Let G be a non-empty goal $\leftarrow C_1, \dots, C_k$ and S be a clause $A \leftarrow B_1, \dots, B_m$. Then the goal G' is obtained by *SLD-resolution* from G and S, if:

1. $Var(G) \cap Var(S) = \emptyset$.

2. C_i and A are unifiable, where C_i is an atom (called the selected atom) in G, $1 \leq i \leq k$.

3. G' is the goal $\leftarrow C_1\theta, \dots, C_{i-1}\theta,\ B_1\theta, \dots, B_m\theta,\ C_{i+1}\theta, \dots, C_k\theta$, where $\theta = mgu(C_i, A)$.

The choice of the selected atom is performed by what is called a selection rule. G' is called an *SLD-resolvent* of G and S.

Let P be a program and G be a goal. An *SLD-derivation* of $P \cup \{G\}$ is a (finite or infinite) sequence $G_0, G_1, \dots$ of goals such that $G_0 = G$, and each G_{i+1} is obtained by *SLD-resolution* from the goal G_i and a clause of P ($i \geq 0$).

Let P be a logic program and G a goal. An *SLD-tree* of P and G is a tree satisfying the following conditions:

1. Each node of the tree is a (possibly empty) goal.

2. The root node is G.

3. Let $\leftarrow C_1, \dots, C_k$ ($k \geq 1$) be a node in the tree and suppose that C_i is the selected atom. Then for each clause $A \leftarrow B_1, \dots, B_m$, $m \geq 0$, in P such that C_i and A are unifiable with mgu θ, the node has a child $\leftarrow C_1\theta, \dots, C_{i-1}\theta,\ B_1\theta, \dots, B_m\theta,\ C_{i+1}\theta, \dots,\ C_k\theta$.

4. Nodes which are empty goals have no children.

To every branch of an *SLD-tree* there corresponds an *SLD-derivation*.

A program P is terminating with respect to a goal G, if the *SLD-tree* of P and G is finite.

Each program P has a corresponding permitted set of goals, which is denoted by $\Delta(P)$. P_1 and P_2 programs are Δ-*equivalent*, if $\Delta(P_1) = \Delta(P_2)$ and for any goal $G \in \Delta(P_1)$, $P_1 \models G$, if and only if $P_2 \models G$ [9].

We will denote by Π_P the set of predicate symbols used in logic program P.

3. FSF Logic Programs. For an FSF program P, a goal is permitted, if it uses predicate symbols only from Π_P.

We will say that atom A *precedes* atom B ($A \preccurlyeq B$), if there exists a substitution θ such that $A\theta = B$. It is easy to see that the relation $\preccurlyeq$ is transitive and reflexive.

We will say that atoms A and B are *congruent* ($A \equiv B$), if $A \preccurlyeq B$ and $B \preccurlyeq A$. It is easy to see that the relation $\equiv$ is an equivalence relation. Congruent atoms are considered identical.

Definition 3.1. For a clause S from logic program P we define a set of atoms $H(P,S) = \{A\theta | A$ is the head of S and the substitution θ uses constants only from program $P\}$.

Lemma 3.1. For a clause S from FSF program P

$$\overline{\overline{H(P,S)}} = \begin{cases} \prod_{j=1}^{k}(l+j), & \text{if} \quad k>0, \\ 1, & \text{if} \quad k=0, \end{cases}$$

where k ($k \geq 0$) is the number of distinct variables in the head of S, and l is the number of distinct constants used in program P.

Proof. Denote by A the head of the clause S. Then $H(P,S)$=A, when $k = 0$. Thus, $\overline{\overline{H(P,S)}} = 1$. Now let us assume that $Var(A) = \{x_1,\dots,x_k\}$, $k > 0$. The corresponding term for the variable x_1 in substitution θ is a constant from P or a variable. The number of these terms is $l+1$. The corresponding term for the variable x_2 in substitution θ is a constant from P or a variable used for x_1 or another variable. So, the number of these terms is $l+2$. And for $x_3,\dots,x_k$ the number of the corresponding terms will be $l+3,\dots,l+k$.

As a result, the number of distinct substitutions for atom A, which use functional symbols only from program P will be $\prod_{j=1}^{k}(l+j)$.

Definition 3.1. For a logic program P we define a set of atoms $H(P)$ as follows: $H(P)$=$H(P, S_1) \cup H(P, S_2) \cup \dots \cup H(P, S_n)$, where $S_1, S_2, \dots, S_n$ are the clauses of P.

It is easy to see that $\overline{\overline{H(P)}} \leq \sum_{i=1}^{n} \overline{\overline{H(P,S_i)}}$.

We denote a tuple of elements $\langle d_1,\dots,d_k\rangle$, $k \geq 0$, by $\overline{d}$ and write $d_i \in \overline{d}$, if d_i is the i-th element of the tuple $\overline{d}$.

Definition 3.2 (Trans2 transformation). Let P be an FSF program. For the program P we construct a program P' as follows:

1. instead of every clause S of the form $p(\overline{t}) \leftarrow p_1(\overline{t_1}),\dots,p_m(\overline{t_m}) \in P$, $m \geq 0$, we define a clause:

$$p^T(\overline{t}, s(z)) \leftarrow p_1^T(\overline{t_1}, z),\dots,p_m^T(\overline{t_m}, z), \text{ called } Trans(S);$$

2. for every predicate symbol $p \in \Pi_P$ we define a clause as follows:

$$p(x_1, \dots, x_k) \leftarrow p^T(x_1,\dots,x_k, s^h(0)), \text{ called } Depth(p),$$

where $h = \overline{\overline{H(P)}}$, $\{z\} \cap Var(S) = \emptyset$, the variables $x_1,\dots,x_k$ are distinct, s is a functional symbol of arity 1, p^T, $p_i^T \notin \Pi_P$ $(0 \leq i \leq m)$, k is the arity of p, and 0 is a constant.

From the 1st point of $Trans2$, for each clause of program P a new clause is defined, which has the same structure as the original one, but with the new predicate symbols having one more arity. The last parameters of the atoms defined with new predicate symbols are used for defining levels. These last parameters are added as constraints. From the 2nd point of $Trans2$, for each clause of program P a new rule is defined, which initializes the level of the head of that clause.

To prove the termination of the transformed programs the following notions, introduced by Bezem [6], will be needed.

Definition 3.3 (level mapping). A *level mapping* is a function $||: B \to N$ from the Herbrand base to the set of natural numbers N. For an atom $A \in B$, $|A|$ denotes the level of A.

A level mapping is only defined for ground atoms. The next definition extends the mapping to cover non-ground atoms. We denote by $ground(A)$ the ground instances of the atom A.

Definition 3.4 (bounded atom). An atom A is bounded *wrt* $||$, if there exists $k \in N$ such that for every $A^{'} \in ground(A)$, $|A^{'}| \leq k$. If A is bounded, then $|A|$ denotes the maximum value of $||$ takes on $ground(A)$.

Definition 3.5 (bounded goal). A goal G of the form $\leftarrow C_1, \dots, C_k$ is bounded (*wrt* $||$), if every C_i is bounded (*wrt* $||$). If G is bounded, then $|G|$ denotes the (finite) multiset consisting of the natural numbers $|C_1|, \dots, |C_k|$.

Level mappings are used to prove termination in the following way. Let $G = = G_0, G_1, G_2, \dots$ be the goals in an SLD-derivation and $||$ a level mapping. Given that G is bounded *wrt* $||$ and $|G_{i+1}|$ smaller than $|G_i|$ for all i, we can deduce that the sequence $G_0, G_1, G_2, \dots$ is finite by the well-foundedness of the natural numbers. This idea is quite old and originates from mathematical logic. The well-founded ordering we shall use is called the *multiset ordering* [10]. The multiset ordering over N is an ordering of finite multisets of natural numbers such that X is smaller than Y $(X < Y)$, if X can be obtained from Y by replacing one or more elements in Y by any (finite) number of natural numbers, each of which is smaller than one of the replaced elements. To prove the goal ordering property, that $|G_{i+1}| < |G_i|$ for all i and for all possible SLD-derivations, Bezem introduced the class of recurrent programs [6], where the goal ordering property is always satisfied.

Definition 3.6 (recurrency). A clause $A \leftarrow B_1, \dots, B_m$ is recurrent (wrt $||$), if for every grounding substitution θ, $|A\theta| > |B_i\theta|$ for all $i \in \{1, \dots, m\}$. A program P is recurrent (wrt $||$), if every clause in P is recurrent (wrt $||$).

L e m m a 3 . 2 [6]. Let P be a logic program, which is recurrent with respect to a *level mapping* $||$. Let G be a bounded goal and $G^{'}$ an SLD-resolvent of G and a clause from P. Then:

1. the goal $G^{'}$ is bounded;

2. the multiset $|G^{'}|$ is smaller than $|G|$ in the multiset ordering.

Corollary. Every SLD-derivation of a recurrent program and a bounded goal is finite.

Proof. Immediate, since the multiset ordering over N is well-founded [10, 11].

A norm is a mapping from ground terms to natural numbers. We will use the term-size norm $||_{term-size}$: $U \to N$ from the Herbrand universe to the natural numbers defined as follows: $|f(t_1,\dots,t_n)|_{term-size} = \begin{cases} n+\sum_{i=1}^{n}|t_i|_{term-size}, & \text{if} \quad n>0, \\ 0, & \text{if} \quad n=0. \end{cases}$

For example, $|s\big(s(0)\big)|_{term-size} = 1+1+0 = 2$.

Theorem 3.1 (Termination). Let P be an FSF program and $P^{'}$ be a program obtained from P by $Trans2$ transformation. Then the SLD-tree of $P^{'}$ and any goal $G \in \Delta(P)$ is finite.

Proof. Since SLD-trees are finitely branching, by Konig's Lemma, "SLD-tree of P' and G is finite" is equivalent to statement that the every SLD-derivation for $P^{'}$ and G is finite. It follows from the Corollary that for proving the finiteness of any SLD-derivation of $P^{'}$ and G it is enough to define level mapping function $||$ and to show that $P^{'}$ is recurrent and G is bounded *wrt* that function.
For an atom $A \in B$, let us define the level mapping function $||$ as follows:

a) $|A| = |\tau|_{term-size}$, if A has a form $p^T(\bar{t},\tau)$, where $p^T \in \Pi_{P'}$ and $p^T \notin \Pi_P$,

b) $|A| = h+1$, in other cases, where $h = \overline{\overline{H(P)}}$.

Let us prove the recurrency of each clause $S \in P^{'}$ *wrt* level mapping:

1. Suppose S is obtained by the 1st point of $Trans2$ transformation and has a form $p^T(\bar{t},s(z)) \leftarrow p_1^T(\overline{t_1},z),\dots,p_n^T(\overline{t_n},z)$. Let $i \in \{1,\dots,n\}$ and θ is a grounding substitution for S. $|p^T(\bar{t},s(z))\theta| = |s(z)\theta|_{term-size} = |z\theta|_{term-size}+1 > |z\theta|_{term-size} =$ $= |p_i^T(\bar{t}_i,z)\theta|$.

2. Suppose S is obtained by the 2nd point of $Trans2$ transformation and has a form $p(x_1,\dots,x_k) \leftarrow p^T(x_1,\dots,x_k,s^h(0))$. Let θ be a grounding substitution for S. $|p(x_1,\dots,x_k)\theta| = h+1 > h = |s^h(0)|_{term-size} = |p^T(x_1,\dots,x_k,s^h(0))\theta|$.

It follows that all clauses of P are recurrent, and hence P is recurrent. Now, let us consider the boundedness of the goal G. Let G be a goal $\leftarrow C_1,\dots,C_m$, $m \geq 1$. According to Definition 3.5, to prove the boundedness of G we should prove the boundedness of every atom C_i, $i \in \{1,\dots,n\}$. Let θ be a grounding substitution for C_i. Then for every i, we have $|C_i\theta| = h+1$. Thus, C_i is bounded and $|C_i| = h+1$.

Having obtained a terminating program, we need to prove that the logical semantics of the transformed program coincides with that of the original one. In order to talk about the Δ-equivalence of original and transformed programs, the standard $T_P : 2^B \to 2^B$ [7] is introduced, which is defined as follows: $T_P(I) = \{A\theta \mid \theta$ is grounding substitution for $A \leftarrow B_1,\dots,B_m \in P$ and $B_1\theta,\dots,B_m\theta \in I\}$.

Define $T_P^k,\ k \geq 0,$ as follows:

1. $T_P^0 = \emptyset$;

2. $T_P^k = T_P(T_P^{k-1}),\ k \geq 1.$

The function T_P has a least fixpoint, denoted by T_P^{fix}, defined as

$$T_P^{fix} = \sup\{T_P^k \mid k \geq 0\}.$$

Lemma 3.3. Let P be an FSF program and $p(\overline{\tau}) \in B$. Then if $P \models p(\overline{\tau})$, it follows that there exists $k \leq h$ such that $T_P^k \models p(\overline{\tau})$, where $h = \overline{\overline{H(P)}}$.

Proof. From definitions of $H(P)$ and T_P, it can be proved that $T_P^h = T_P^{fix}$. So, there exists $k \leq h$ such that $T_P^k = T_P^{fix}$. As T_P^{fix} coincides with the least Herbrand model for P [7], it follows from $P \models p(\overline{\tau})$ that $T_P^{fix} \models p(\overline{\tau})$. The result then follows.

Lemma 3.4. Let P be an FSF program and P' be a program obtained from P by $Trans2$. Then if $T_{P'}^k \models p^T(\overline{\tau}, s^n(0))$, it follows that $T_{P'}^k \models p^T(\overline{\tau}, s^{n+1}(0))$, where $p^T(\overline{\tau}, s^n(0)) \in B,\ p \in \Pi_P,\ n,k \geq 0$.

Proof. The proof follows straightforwardly by analysing the structure of P'.

Lemma 3.5. Let P be an FSF program and P' be a program obtained from P by $Trans2$. Then if $T_P^k \models p(\overline{\tau})$ and $k \leq h$, it follows that:

a) $T_{P'}^k \models p^T(\overline{\tau}, s^k(0))$;

b) $T_{P'}^{k+1} \models p(\overline{\tau})$,.

where $p(\overline{\tau}) \in B,\ p \in \Pi_P$, $k \geq 0$ and $h = \overline{\overline{H(P)}}$.

Proof. We prove induction. For $k = 0$, $T_P^k = \emptyset$ and the result obviously holds. Thus, consider $k \geq 1$. Suppose $T_P^k \models p(\overline{\tau})$. Then a grounding substitution θ exists for a clause S of the form $p(\bar{t}) \leftarrow p_1(\bar{t}_1), \ldots, p_m(\bar{t}_m) \in P$ such that $p(\bar{t})\theta = p(\overline{\tau})$ and $p_1(\bar{t}_1)\theta, \ldots, p_m(\bar{t}_m)\theta \in T_P^{k-1}$. By induction, it follows that for every $i \in \{1, \ldots, m\}$, $T_{P'}^{k-1} \models p_i^T(\bar{t}_i, s^{k-1}(0))\theta$. Because $Trans(S)$ is $p^T(\bar{t}, s(z)) \leftarrow p_1^T(\bar{t}_1, z), \ldots, p_m^T(\bar{t}_m, z)$ and $\sigma = \theta \cup \{z/s^{k-1}(0)\}$ is a grounding substitution for $Trans(S)$, it follows that $T_{P'}(T_{P'}^{k-1}) \models p^T(\bar{t}, s(s^{k-1}(0)))\sigma$. Hence, $T_{P'}^k \models p^T(\overline{\tau}, s^k(0))$. Since $k \leq h$, (Lemma 3.4) it follows from $T_{P'}^k \models p^T(\overline{\tau}, s^k(0))$ that $T_{P'}^k \models p^T(\overline{\tau}, s^h(0))$. As $Depth(p)$ is $p(x_1, \ldots, x_k) \leftarrow p^T(x_1, \ldots, x_k, s^h(0))$, it follows that $p(\overline{\tau}) \in T_{P'}(T_{P'}^k)$, so $T_{P'}^{k+1} \models p(\overline{\tau})$.

Lemma 3.6. Let P be an FSF program. P' is a program obtained from P by $Trans2$. Then, $P \models p(\overline{\tau})$, if and only if $P' \models p(\overline{\tau})$, for all $p \in \Pi_P$, $p(\overline{\tau}) \in B$.

Proof.

a) Suppose $P' \models p(\overline{\tau})$. As $Trans2$ saves the structure of the original program by adding only constraints, we straightforwardly get that $P \models p(\overline{\tau})$.

b) Suppose $P \models p(\overline{\tau})$. There exists $k \leq h$ such that $T_P^k \models p(\overline{\tau})$ (Lemma 3.3). Hence, $T_{P'}^{k+1} \models p(\overline{\tau})$ (Lemma 3.5). As $T_{P'}^{k+1} \subset T_{P'}^{fix}$, it follows that $T_{P'}^{fix} \models p(\overline{\tau})$. Hence, $T_{P'}^{fix}$ coincides with the least Herbrand model for P', and, consequently, $P' \models p(\overline{\tau})$ [7].

Theorem 3.2 (Equivalence). Let P be an FSF program and P' be a program obtained from P by $Trans2$ transformation. Then, for any goal $G \in \Delta(P)$, $P \models G$, if and only if $P' \models G$.

Proof. Immediate from Lemma 3.6.

In other words, the transformed program and the original one are Δ-equivalent.

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