PROCEEDINGS OF THE YEREVAN STATE UNIVERSITY

Physical and Mathematical Sciences 2017, **51**(1), p. 71–76

Physics

STRANGE STARS AT VACUUM PRESSURE DEPENDENT ON QUARK DENSITY

Yu. L. VARTANYAN [*], A. K. GRIGORYAN [**], H. A. SHAHINYAN [***]

Chair of Wave Process Theory and Physics YSU, Armenia

Equation of state of strange quark matter has been studied in the framework of MIT bag model, when vacuum pressure B depends on concentration of baryons n. The actuality of such studies is conditioned by the increasing of quark matter density from surface to star center. In the literature there exist different representations of function $B(n)$. In the present work Gaussian parametrization is used, which is based on the idea of existence of asymptotic limiting value of this parameter. For four groups of parameters the equations of state of quark matter were determined. The main integral parameters of star configurations were obtained by numerically integrating of star equilibrium equations (the TOV equation). In the considered case it turns that when vacuum pressure dependence on concentration of baryons is taken into account, configurations of strange stars have maximal masses less than two solar masses.

Keywords**:** strange stars, strange quark matter, vacuum variable pressure.

Introduction. Study of astrophysical effects of superdense stars, consisted of strange quark matter (SQM), so called strange stars, is one of prioritized problems of temporary theoretical astrophysics. For studies such superdense stars (SS), it is important to examine the dependence of their mass M, and other integral parameters (radius, total baryon number, binding energy, moment of inertia) on the central energy density ρ_c (or central pressure P_c). Quark matter properties have been studied for decades based on phenomenological models [1–7].

To describe properties of SQM, one of generally accepted models is the bag model (BM), elaborated in Massachusetts Technological Institute (MIT) [4, 5]. By choosing BM the equation of state of SQM is determined. BM is characterized by three phenomenological parameters: bag constant B (vacuum pressure), quark-gluon interaction constant α_c and strange quark mass m_s. Numeric values of these parameters are known with high uncertainty, which makes necessary the study of principally different variants of SQM. For various groups of these constants the equations of state are obtained that lead to realization of both strange stars and neutron stars with quark core, where quark matter is in phase balance with nucleon one. The latter's are studied in details in [8–11]. In [12, 13] the groups of values of bag parameters are identified, using of which in the equation of state leads to the maximum mass of equilibrium quark configurations, which are bigger than the mass of known radio pulsar PSR $J0348+0432 : M_{\max} > 2.01\, M_{\odot}$.

This work is devoted to observation of the problem of integral parameters of self-bounded SS and to study of SQM in the framework of the bag model, when vacuum pressure

[*] E-mail: yuvartanyan@ysu.am [**]agrigoryan@ysu.am [***]hasmik.shahinyan@gmail.com

depends on concentration of baryons $B(n)$. The actuality of such studies is explained by the increasing of quark matter density from surface to star center. The work consists of three parts. In the part "Equation of State" thermodynamics and equation of state of SQM are discussed in frames of the bag model at vacuum variable pressure. In the present work for dependence of $B(n)$ Gaussian parametrization is used [14], which is constructed on the idea of existence of asymptotic limiting value of this parameter. Four groups of the bag parameters are observed. Calculations are carried out for the following values of the bag parameters: $m_s = 95$ and 150 MeV and $\alpha_c = 0.01$ and 0.05. Masses of u, d quarks and electrons are neglected due to their smallness. The main thermodynamic values are determined: concentration of baryons n, average energy per baryon ε, pressure P and energy density ρ. In the part "Results of Calculations" integral parameters of star configurations are discussed at variable pressure of vacuum. Dependence of star total mass on radius at fixed and variable pressure of vacuum are presented. Results of calculations show that in the case, when vacuum pressure dependence on concentration of baryons is taken into consideration, configurations of SS have maximal masses less than two solar one.

Equation of State. It has been shown in [15–17], that SQM consisted of u, d, s quarks even at zero pressure can be energetically more preferable, than nonstrange quark matter (NQM) consisted of u, d quarks and the matter in atomic nuclei N. Witten showed [17], that neglecting the interaction between quarks (the ideal gas), the ratio of chemical potentials per baryon in ultra-relativistic SQM and NQM at the same pressure is equal to 0.89. For SQM at zero pressure it results in energy per baryon, which is less by 84.4 MeV than neutron masses. This difference at accounting of SQM ($m_s c^2 = 200$ MeV), remaining significant, decreases and reaches up to 30.4 MeV. Moreover, at zero pressure, because of the absence of strange quarks, the state of SQM vacant-baryonic matter is in nucleon (nuclear) state N, which is energetically more favorable than NQM. Neutron chemical potential approaches to the chemical potential per baryon in NQM at some critical pressure $P = P_0$, i.e. $\mu(N) = \mu(\text{NQM})$ and for $P > P_0$, $\mu(\text{NQM}) < \mu(N)$. u and d quarks are released from nucleons, i.e. there is a transition from N to NQM state. But since the relation of chemical potentials SQM and NQM is equal to 0.89, in the result of weak interaction there is phase transition from NQM to SQM or ultimately, a transition from N to SQM.

It has been shown in [18], that if the bag model is taken for quarks [4], the strange quark model consisted of almost equal amounts of u, d, s quarks and small additive of electrons, which contribute to their electrical neutrality, for certain values of the bag constants (B, α_c, m_s) lead to the case, when the average energy ε per baryon depending on specific volume of baryons $(1/n)$ can have as a positive, so a negative local minimums ($\varepsilon_{\min} \lessgtr 0$), which in its turn leads to two alternative possibilities.

In the first case a phase balance between strange quark and nucleon-hyperon (core) matters is realized, i.e. a simultaneous coexistence of two phases takes place. Superdense stars, corresponding to such equation of state, are accepted to call hybrid stars [10, 11].

In the second case, when $\varepsilon < 0$, a phase transition from nucleon-nuclear state to quark state is not possible. Quark matter can be found in a self-bound state, so that it is possible for self-confining, so called "strange stars" to exist [19, 20]. These stars can exist in the absence of gravitation as well. This limits the maximum mass of these configurations, which as in the case of neutron stars, turns to be $\sim 2M_\odot$.

Let observe the equation of state of strange quark matter, when vacuum pressure depends on concentration of baryons. According to the bag theory SQM consists of degenerated Fermi gas of u, d, s quarks and electrons. It is assumed that quarks and gluons are enclosed in the area of dimension called "bag", where they are retained by vacuum pressure B bag constant. Accounting of bag pressure dependence on baryon density represents a special interest. In the literature there exist various representations for $B(n)$ function [14, 21]. In this work one of effective parameterizations "Gaussian parametrization" is used, based on the idea of existence of asymptotic limiting value of this parameter at $n \to \infty$:

$$B(n) = B_\infty + (B_0 - B_\infty)\exp\left\{ -\gamma(n/n_0)^2 \right\}. \quad (1)$$

Value of parameter is γ and depends on parameter B_0. It should be mentioned that the precise value B_0, $B_0 = B(n = 0)$ is not so important, since at low densities, in any case, the matter is in nucleon phase. In [14, 22] two variants are observed: $B_0 = 200\ MeV/fm^3$, $\gamma = 0.14$ and $B_0 = 400\ MeV/fm^3, \gamma = 0.17$. n_0 is the nuclear number density. For determination of value of B_∞ parameter in [14] a formula is suggested, where energy value of phase transition is used, obtained from experiments. Table for values of B_∞ parameter is provided. It should be mentioned that $B \underset{n\to\infty}{\longrightarrow} B_\infty$. Calculations show that B_∞ can accept values in diapason $14.3 - 55.4\ MeV/fm^3$ [14] and choice of value in this diapason does not change the main result.

Four groups of the bag parameters are presented with the following values of constants: $B_0 = 200$, $B_\infty = 38\ MeV/fm^3, \gamma = 0.14$, $n_0 = 95\ fm^{-3}$. For the first two models the mass of strange quark is accepted to be equal to current mass of quark $m_s = 95\ MeV$, $\alpha_c = 0.01;\ 0.05$. For the second group the strange quark mass is equal to 150 MeV. As in the previous cases, masses of and quarks and electrons are neglected due to smallness (Tab. 1). Dependence of vacuum pressure on density for the Model 2 is presented in Fig. 1. It is obvious from the figure that at low densities, when the concentration value $n \to 0$, the vacuum pressure value $B \to B_0$. Furthermore, in this case the matter is in nucleon phase. When $B \underset{n\to\infty}{\longrightarrow} B_\infty$, which means that parameter B has some limiting value. It is obvious from the figure that the real diapason of changes of B varies between $38 - 60\ MeV/fm^3$ for SQM, which corresponds to $n = 0.6 - 0.9\ fm^{-3}$.

Table 1

Parameters of bag group for 4 models of SQM

Model 1	Model 2	Model 4	Model 5
$\alpha_c = 0.01$	$\alpha_c = 0.05$	$\alpha_c = 0.01$	$\alpha_c = 0.05$
$m_S = 95\ MeV$	$m_S = 95\ MeV$	$m_S = 150\ MeV$	$m_S = 150\ MeV$

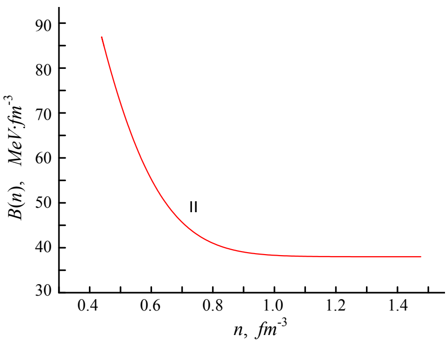

Fig. 1. Vacuum pressure dependence $B(n)$ on baryon density n for II model.

Let observe equation of state of SQM. The β equilibrium condition implies the following relation among the quarks:

$$\mu_d = \mu_u + \mu_e, \quad \mu_d = \mu_s \equiv \mu, \quad (2)$$

where μ_i $(i = u,\ d,\ s,\ e)$ are chemical potentials of $u,\ d,\ s$ quarks and electrons. Eq. (2) have to be supplemented by the condition of electrical neutrality,

$$2n_u/3 - n_d/3 - 3/n_s - n_e = 0, \quad (3)$$

where $n_i(\mu_i) = \partial\Omega_i/\partial\mu_i$ are the concentrations of corresponding components. Ω_i are thermodynamic potentials obtained in linear approximation by α_c $(i = u, d, s, e)$ [18], determining SQM properties. These quantities depend on α_c and μ_i, Ω_s additionally depends on m_s and ρ_R which are points of renormalization for SQM and is taken to be 313 MeV [18]. Eq. (2) and Eq. (3) can be used to obtain the main thermodynamic quantities (the pressure P, energy density ρ, baryons concentration n, and baryon chemical potential μ_Q) as function of the only independent parameter μ for SQM:

$$P = -\sum_i \Omega_i(\mu) - B(n) + n\partial B/\partial n, \quad \rho = \sum_i (\Omega_i + \mu_i n_i) + B, \tag{4}$$

$$n = (n_u + n_d + n_s)/3, \quad \mu_Q(P) = (\rho(P) + P)/n(P). \tag{5}$$

For variable vacuum pressure due to additional member, the total pressure changes and takes the form of first Eq. (4) [23].

Results of Calculations. The main parameters of spherically symmetric static SS, as in the case with fixed vacuum pressure, are determined by numerical integration of a system of relativistic equations for stellar equilibrium, that is the TOV equations [24, 25]. We do not give these equations here, but note only that the compact form we are using, is given in [26]. The star's radius $P(R) = 0$, gravitational mass $M = (4\pi/c^2)\int_0^R \rho r^2 dr$, rest mass $M_0 = (4\pi m_0)\int_0^R n r^2 \exp(\lambda/2) dr$, proper mass $M_p = (4\pi/c^2)\int_0^R \rho r^2 \exp(\lambda/2) dr$ and red shift $Z_S = (1 - 2GM/c^2 R)^{-1/2} - 1$ from star's surface are calculated as function of ρ_c. Here $\exp(\lambda)$ is the radial component of the metric tensor. It should be mentioned that minimum value of the average energy per baryon for cases with $m_s = 150$ MeV has positive sign. It means that such models with $\varepsilon_{\min} > 0$, lead to existence of hybrid stars with strange quark core. Moreover, for the cases $m_S = 95$ MeV, the average energy per baryon has negative minimum. It should be mentioned as well that values of the minimum energy per baryon significantly depend on α_c. Groups of the bag parameters for Models 1 and 2 result in realization of self-bound SS.

Table 2

Integral parameters of configuration of maximum masses for different models of SQM

Models of SQM	$\varepsilon_{\min}$, MeV	$\frac{n_{\min}}{n_0}$	$\rho_S \cdot 10^{-15}$, g/cm^3	$\frac{M_{\max}}{M_\odot}$	$\frac{M_0}{M_\odot}$	$\frac{M_p}{M_\odot}$	$\rho_c \cdot 10^{-15}$, g/cm^3	Z_S	R, km
1	−17.35	3.792	0.924	1.875	2.398	2.418	2.441	0.543	9.533
2	−10.84	3.777	0.927	1.872	2.377	2.414	2.443	0.542	9.517
3	4.964	3.799	0.943	1.811	2.252	2.325	2.521	0.530	9.320
4	10.990	3.815	0.957	1.808	2.235	2.322	2.517	0.531	9.306

Values of the average energy per baryon $\varepsilon_{\min}$, which correspond to value $n_{\min}$, are presented in Tab. 2. For each equation of state the values of the density on surface ρ_s also presented in the Table. The baryon concentration and energy density for a given equation of state are determined by the values of $n_{\min}$ and ρ_s during the transition to SQM. From the presented Table data, it follows that on star surface the transition to strange quark matter occurs at the following order of baryon concentration $n_{\min} = 3.8\, n_0$, where $n_0 = 0.15\ fm^{-3}$ is nuclear concentration. It should be noted that at fixed value of vacuum pressure, the transition to SQM takes place at densities significantly lower than the value of ρ_s for $M_{\max}$ of neutron stars and relation $n_{\min}/n_0$ for SS is in order of ~ 1.7, i.e. $n_{\min} < 2\ n_0$ [12, 13]. It means that at fixed value of B (at small baryonic concentrations) for SS the transition to strange quark phase occurs earlier than in the case with variable B. For observed models of SQM, the equations of state lead to configurations with maximum masses less than pulsar mass PSR $J0348+0432: M_{\max} < 2.01\ M_\odot$. Though for such configurations the values of masses, radius total number of baryons as well as red shift from SS surface were calculated depending on central density energy ρ_s. Maximum mass of such configurations is in order of $1.9\ M_\odot$, which is less compared to recently registered radio pulsars [27] and [28]. Radius of such stars is in order of 9.5 *km*.

Equation of state for III and IV groups of the bag parameters leads to positive local minimum of energy per baryon of SQM: $\varepsilon_{\min} > 0$, which results in hybrid stars. It follows from Tab. 2, that the mass of the strange core of such stars can be in order of $1.8\ M_\odot$, which is less than our known double radio pulsar mass by almost $0.21\ M_\odot$ [27]. Furthermore for hybrid stars in central core (consisted of SQM) almost 90% of whole mass is concentrated. 10% of mass composes crust from neutron star matter. If the crust mass is added to value $M_{\max}$, obtained for III and IV configurations, at given description of SQM such hybrid stars, having a mass in order of $2.01\ M_\odot$, can be possible candidates for the aforementioned radio pulsars. Similar calculations may be useful at investigation of structure of hybrid stars with maximal masses [29].

Dependence of rest mass of strange stars on radius are represented in Fig. 2. The first curve corresponds to the case, when vacuum pressure is fixed: $B = 60\ MeV/fm^3; \alpha_c = 0.05$; $m_S = 95\ MeV$. This group of the bag parameters provides a self-bound state of strange quark matter, which leads to existence of SS with maximum mass $1.89\ M_\odot$. The second curve corresponds to the second group of the bag parameters presented in Tab. 1. Choice of value of parameter $B = 60\ MeV/fm^3$ for fixed B corresponds to vacuum pressure value in point, where the average energy per baryon is minimum: $\varepsilon = \varepsilon_{\min}$. It is obvious from the figure that for the given value of the rest mass, i.e. for the same value of number of baryons, radius of the model with fixed B is bigger than that for the model with variable $B(n)$.

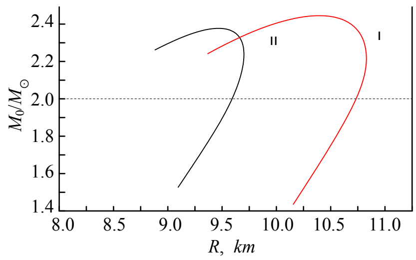

Fig. 2. Dependence of rest mass $M_0/M_\odot$ on radius R for two configurations of *SS*.

Conclusion. In the present paper the equation of state of SQM is studied in the frames of the bag model, when vacuum pressure depends on baryons concentration. Gaussian parametrization for $B(n)$ is used. Groups of the bag parameters are determined that result in existence of self-bound SS. For 4 pairs of parameters α_c and m_S, dependence of pressure and energy per baryon ε on concentration of baryons are determined in frames of MIT bag theory with parameter $B = B(n)$ dependent on baryons concentration. It is shown that this energy is minimal at $n_{\min} = 3.7n_0$, where $n_0 = 0.15\ fm^{-3}$. Integral parameters of SS are determined. It is shown that in the case with variable pressure of vacuum, groups of SQM parameters do not provide $M_{\max} \geq 2.01\ M_\odot$, and are in order of $1.9\ M_\odot$.

Received 30.11.2016

REFERENCES

1. **Buballa M.** NJL-Model Analysis of Dense Quark Matter. // Physics Reports, 2005, v. 407, p. 205–376.
2. **Nambu Y., Jona-Lasinio G.** Dinamical Model of Elementary Particles Based on an Analogy with Superconductivity. // Physical Review, 1961, v. 122, p. 345–358.
3. **Nambu Y., Jona-Lasinio G.** Dinamical Model of Elementary Particles Based on an Analogy with Superconductivity. II. // Physical Review, 1962, v. 124, p. 246–254.
4. **Chodos A., Jaffe R.L., Johnson K., Thorne C.B., Weisskopf V.F.** A New Extended Model of Hadrons. // Phys. Rev. D, 1974, v. 9, p. 3471.
5. **Chodos A., Jaffe R.L., Johnson K., Thorne C.B.** Baryon Structure in the Bag Theory. // Phys. Rev. D, 1974, v. 10, p. 2599.
6. **Chakrabarty S., Raha S., Sinha B.** Strange Quark Matter and the Mechanism. // Phys. Lett. B, 1989, v. 113, p. 229.
7. **Benvenuto O.G., Lugones G.** Strange Matter Equation of State in the Quark Mass-Density-Dependent Model. // Phys. Rev. D, 1995, v. 51, p. 1989–1993.
8. **Itoh N.** Hydrostatic Equilibrium of Hypothetical Quark Stars. // Prog. Theor. Phys., 1970, v. 44, p. 291.
9. **Baym G., Chin S.** Can a Neutron Star be a Giant MIT Bag? // Phys. Lett. B, 1976, v. 62, p. 241.
10. **Alaverdyan G.B., Harutyunyan A.R., Vartanyan Y.L.** Neutron Stars with a Quark Core. I: Equations of State. // Astrophysics, 2003, v. 46, p. 361.
11. **Alaverdyan G.B., Harutyunyan A.R., Vartanyan Y.L.** Neutron Stars with a Quark Core. II: Basic Integral and Structural Parameters. // Astrophysics, 2004, v. 47, p. 52.
12. **Vardanyan Y.L., Grigoryan A.K., Shahinyan H.A.** Maximal Mass of Strange Stars and Pulsar-swith More Precisely Measured Masses. // Astrophysics, 2015, v. 58, p. 297–310.
13. **Vardanyan Y.L., Grigoryan A.K., Shahinyan H.A.** Pulsar PSR J0348-0432 and Strange Stars. // Letters in Astronom. Journal, 2015, v. 41, p. 1–8.
14. **Burgio G.F., Baldo M., Sahu P.K., Schulze H.-J.** Hadron-Quark Phase Transition in Dense Matter and Neutron Stars. // Phys. Rev. C., 2002, v. 66, p. 025802.
15. **Bodmer A.R.** Collapsed Nuclei. // Phys. Rev. D, 1971, v. 4, p. 1601.
16. **Terazawa H.** INS-Report-338. // INS, Univ. of Tokyo, 1979.
17. **Witten E.** Cosmic Separation of Phases. // Phys. Rev. D, 1984, v. 30, p. 272.
18. **Jaffe R.L., Farhi E.** Strange Matter. // Phys. Rev. D, 1984, v. 30, p. 2379.
19. **Alcock C., Olinto A.** Exotic Phases of Hadronic Matter and Their Astrophysical Application. // Ann. Rev. Nucl. Part. Sci., 1988, v. 38, p. 161–184.
20. **Benvenuto O.G., Horvarth J.E., Vucetich H.** Strange-Pulsar Models. // Intern. J. of Modern Phys., 1991, v. A6 (27), p. 4769–4830.
21. **Adami C., Brown G.E.** Matter Under Extreme Conditions. // Phys. Rep., 1993, v. 234, p. 1–71.
22. **Burgio G.F., Baldo M., Sahu P.K., Santra A.B., Schulze H.-J.** Maximum Mass of Neutron Stars with a Quark Core. // Phys. Lett. B., 2002, v. 526, p. 19–26.
23. **Zhu M.F., Liu G.Z., Yu Z., Xu Y., Song W.T.** Thermodynamics of Strange Quark Matter with the Density-Dependent Bag Constant. // Science in China Series G, 2009, v. 52(10), p. 1506–1512.
24. **Tolman R.C.** Static Solutions of Einstain's Field Equations for Spheres of Fluid. // Phys. Rev., 1939, v. 55, p. 364.
25. **Oppenheimer J.R., Volkoff G.M.** On Massive Neutron Cores. // Phys. Rev., 1939, v. 55, p. 374.
26. **Vartanyan Y.L.** Superdense Stars Containinig Strange Baryons. // Astrophysics, 2010, v. 53, p. 18.
27. **Antoniadis J., Freire P.C.** et al. A Massive Pulsar in a Compact Relativistic Binary. // Science, 2013, v. 340, p. 1233232.
28. **Demorest P., Pennucci T., Ransom S.M., Roberts M.S.E., Hessels J.W.T.** A Two-Solar-Mass Neutron Star Measured Using Shapiro Delay. // Nature, 2010, v. 467, p. 1081.
29. **Burgio G.F., Baldo M., Chen H., Schulze H.-J.** The Equation of State at Finite Temperature: Structure and Composition of Protoneutron Stars. // J. Phys. Conf. Ser., 2016, v. 665, p. 012062.